ATTENUATION CHARACTERISTICS OF A HELMHOLTZ TYPE OF HYDRAULIC SILENCER WITH HEMISPHERICAL VESSEL SHAPE

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ABSTRACT

Pressure ripple is one of the primary sources of noise in hydraulic circuits, because it transmits throughout a fluid power system and excites the mechanical parts thus generating audible noise. Various types of hydraulic silencers may be installed in hydraulic systems in order to attenuate the pressure ripple. The Helmholtz type of hydraulic silencer is known as one of the most practical silencers on account of simple structure and reasonable price. The maximum attenuation performance can be attained at the resonance frequency in accordance with the principle of the Helmholtz resonator. Therefore, it is extremely important to estimate the resonance frequency precisely at the design stage. It is relatively easy to determine the dimensional specification of a silencer by using the lumped parameter model. However, it is often the case that the estimated value of the resonance frequency differs from the practical value. Consequently, the utilization of distributed parameter model is a better strategy for designing the silencer.

In this study, the attenuation characteristics of the silencer with a hemispherical vessel shape are investigated. A novel distributed parameter model is proposed, which takes into account the propagation of a one-dimensional wave in the radial direction of the spherical vessel. Furthermore, the effectiveness of the theoretical analysis is verified by the experiments which are performed by changing the dimensional specification of the vessel and neck.

KEY WORDS

Noise and vibration, Pressure ripple, Attenuation characteristics, Helmholtz resonator

NOMENCLATURE

$c$ : Speed of sound in hydraulics
$D$ : Diameter of hemispherical vessel
$D_1, D_2$ : Constant of integration
$d$ : Diameter of neck
$d_m$ : Diameter of main line
$f_r$ : Resonance frequency

$fr$ : Resonance frequency

$j$ : imaginary unit

$j_i$ : Spherical Bessel function of first kind of order $i$
$K$ : Bulk modulus of oil
$l$ : Length of neck

$l_n$ : Length of neck considering open end correction

$n_i$ : Spherical Bessel function of second kind of order $i$
$P$ : Pressure ripple in Laplace domain
$p$ : Pressure ripple
Q : Flow ripple in Laplace domain
q : Flow ripple
s : Laplace operator
\( T_{\text{N}} \) : Transfer matrix of neck
\( T_{\text{S}} \) : Transfer matrix of silencer
\( T_{\text{V}} \) : Transfer matrix of vessel
\( TL \) : Transmission loss
\( TL_r \) : Transmission loss at resonance frequency
\( v_r \) : Fluid velocity for radial direction
\( Z_c \) : Characteristics impedance of main line
\( Z_n \) : Characteristics impedance of neck
\( \beta \) : Complex coefficient
\( \varepsilon \) : Dimensional index on neck
\( \nu \) : Kinematic viscosity of oil
\( \rho \) : Density of oil
\( \xi \) : Complex coefficient for unsteady viscous friction effect

INTRODUCTION

It is defined by ISO 5598 that pressure ripple is a fluctuating component of pressure in the hydraulic fluid caused by interaction of the source flow ripple with the system. In order to prevent this, various types of hydraulic silencers are employed in the field of industrial and constructional machinery. The hydraulic silencers based on the principle of the Helmholtz resonator have the benefit of simple construction and excellent cost performance. However, the silencer has the drawback that it is effective only in the narrow-band range around the fundamental frequency. Therefore, it is very important to predict the appropriate attenuation characteristics by means of the mathematical model.

In the previous papers on Helmholtz type of hydraulic silencers [1], [2], the authors have revealed the influence that the geometry of the cylindrical vessel has influence on the attenuation characteristics. In the case that the cylindrical vessel shape is elongate, it is possible to clarify the attenuation characteristics by using the distributed parameter model which was classically introduced by the Navier-Stokes equation for the longitudinal direction in cylindrical coordinates. On the other hand, when the silencer has a flat cylindrical vessel shape, a new distributed parameter model for the radial direction was proposed by our group. It was found that the two distributed parameter models can be used to clarify the attenuation characteristics. In the research reported here, a mathematical model which may be appropriate to a silencer with a hemispherical vessel shape is presented.

The attenuation characteristics of Helmholtz resonators have been sufficiently investigated in the field of acoustics. In the research [3], the analysis of a mathematical model for a spherical vessel shape has been reported. However, this model is not applicable to the hydraulic silencer because the fluid medium of the system is gas. When the working fluid is mineral oil, it is necessary to take viscosity into account. In hydraulic systems, it can be seen that the Helmholtz type of silencer with spherical vessel shape is often employed. However, there is no report on the attenuation characteristics of these kinds of silencer, as far as authors recognize. In engineering applications, it is useful to construct a distributed parameter model for the hydraulic silencer which has spherical vessel shape and neck.

The aim of this study is to examine the attenuation characteristics of hydraulic Helmholtz silencers with hemispherical vessel shapes, since there is no sense to construct a sphere vessel with a source at the center in practice. Firstly, a novel theoretical analysis is presented, which employs a transfer matrix based on the Navier-Stokes equation for the radial direction in spherical coordinates. Secondly, in the view of the attenuation characteristics, the proposed model is compared to the results of the experiments and lumped parameter model.

THEORETICAL ANALYSIS

Distributed Parameter Model

(1) Transmission loss

The Helmholtz type of hydraulic silencer simply consists of a vessel and neck connected with a fluid line. The physical model of the silencer is described as follows. The neck and vessel act as a mass and spring due to the inertia and capacitive effects of the hydraulic fluid respectively. Therefore, this silencer comprises a vibration system with a single degree of freedom in addition to the viscous damping in the neck. Based on the principle of a lumped parameter model, the resonance phenomenon can often be utilized for the attenuation of pressure ripple in hydraulic systems.

Figure 1 shows the structure of the silencer with...
hemispherical vessel shape. The one-dimensional spherical wave propagation which transmits from the center toward the spherical wall of the vessel along the radial direction is most dominant term considering the vessel configuration. It is convenient to use a space polar coordinate whose origin is at the intersection of neck and vessel. As shown in Fig. 1, the coordinate in the radial direction is defined as \( r \). Transmission loss is usually a measure of the attenuation performance of hydraulic silencers. This is defined as the logarithmic ratio of the incident and transmitted pulsation wave energy of a hydraulic silencer under the anechoic termination condition [4]. In the case that the silencer is branched off from the main line, the transmission loss \( TL \) can be expressed by:

\[
TL = 20 \log_{10} \left( \frac{1}{2} + Z_c \frac{T_{S,21}}{T_{S,22}} \right)
\]  

(1)

where \( Z_c \) is the characteristic impedance of the main line, and is obtained from the following equation:

\[
Z_c = \frac{4 \rho \beta}{\pi d_m^2}
\]  

(2)

c, \( \rho \) and \( d_m \) are speed of sound in hydraulic, density of hydraulic and the diameter of main line respectively. In Eq. (1), \( T_{S,21} \) and \( T_{S,22} \) are the elements of transfer matrix of the silencer \( T_S \) as shown in Eq. (3) latter.

The relationship between the pressure ripple and flow ripple at each inlet and outlet of the silencer \( P_{1} , Q_{1} \) and \( P_{2} , Q_{2} \) is expressed by \( T_S \) as follows:

\[
\begin{bmatrix}
P_1 \\ Q_1 
\end{bmatrix} = T_S \begin{bmatrix}
P_1 \\ Q_1 
\end{bmatrix} \begin{bmatrix}
T_{S,11} & T_{S,12} \\ T_{S,21} & T_{S,22}
\end{bmatrix} \begin{bmatrix}
P_2 \\ Q_2 
\end{bmatrix}
\]  

(3)

Once the transfer matrix \( T_S \) is determined from the above equation, the transmission loss can be calculated by using Eq. (1).

(2) **Transfer matrix of vessel**

The complete Navier-Stokes equation for radial direction and the continuity equation considering compressibility in a space polar coordinate are given as the basic equation [5]. In this study, the following four assumptions are made:

(i) The change of all dependent variables in the circumferential direction is negligible due to rotational symmetry.

(ii) Since the velocity \( v_{\theta} \) for the circumferential direction is much smaller than that for the radial direction \( v_r \), the velocity \( v_{\theta} \) may be neglected. This implies that the pressure is constant over the cross section of the spherical area and becomes a function only of radius \( r \) and time \( t \).

(iii) The velocity \( v_r \) for the radial direction is sufficiently slow compared to the speed of sound. Therefore, a term of the convective acceleration may be neglected.

(iv) The velocity \( v_r \) for the radial direction is not affected by the wall viscosity on the bottom surface of the vessel.

With these assumptions, the basic equations are described as follows:

\[
\frac{\partial v_r}{\partial t} = \frac{1}{\rho} \frac{\partial P}{\partial r} + v \left( \frac{4}{3} \frac{\partial^2 v_r}{\partial r^2} + \frac{8}{3r} \frac{\partial v_r}{\partial r} - \frac{8}{3} \frac{v_r}{r^2} \right)
\]  

(4)

\[
\frac{1}{K} \frac{\partial P}{\partial t} + 2 v_r + \frac{\partial v_r}{\partial r} = 0
\]  

(5)

where, \( K \) and \( \nu \) are bulk modulus of elasticity and kinematic viscosity respectively. Applying Laplace transformations to these equations, the pressure ripple \( P_i(s, r) \) and flow ripple \( Q_i(s, r) \) in the Laplace domain are given as follows:

\[
P_i(s, r) = -\frac{K}{2rs} \left[ C_1(r\beta j_0 + 3j_1 - r\beta j_2) + C_2(r\beta n_0 + 3n_1 - r\beta n_2) \right]
\]  

(6)

\[
Q_i(s, r) = 2\pi r^2 (C_1 j_1 + C_2 n_1)
\]  

(7)

where, \( D_1 \) and \( D_2 \) are complex integration constants, and also \( j_i \) and \( n_i \) expressed the spherical Bessel functions of the first and the second kind of order \( i \) \((i=0-2)\). These are described as follows:

\[
j_i = j_i(\beta r)
\]  

(8)

\[
n_i = n_i(\beta r)
\]  

(9)

In these expressions, the complex coefficient \( \beta \) is given as follows:

\[
\beta = j \sqrt{\frac{s}{4} + \frac{K}{\rho^2}}
\]  

(10)

By using the boundary condition at the inner radius \( r_1 = d/2 \) and the outer radius \( r_2 = D/2 \) of the hemispherical vessel, the relationship between the pressure ripple and flow ripple at each radius \( (P_{1,i}, Q_{1,i} \) and \( P_{2,i}, Q_{2,i}) \) is expressed by using a transfer matrix \( T_V \).
\[
\begin{bmatrix} P_{r,1} \\ Q_{r,1} \end{bmatrix} = T_r \begin{bmatrix} P_{r,2} \\ Q_{r,2} \end{bmatrix} = \begin{bmatrix} T_{r,11} & T_{r,12} \\ T_{r,21} & T_{r,22} \end{bmatrix} \begin{bmatrix} P_{r,2} \\ Q_{r,2} \end{bmatrix}
\] (11)

The elements of transfer matrix \( T_r \) are given as follows:

\[
T_{r,11} = \frac{\pi \cos[(\varpi + \eta_1) \beta] + \frac{1}{\eta_1} \sin[(\varpi + \eta_2) \beta]}{\eta_1 \beta}
\] (12)

\[
T_{r,12} = \frac{K \beta}{2\pi \eta_1 \rho_s} \sin[(\varpi + \eta_2) \beta]
\] (13)

\[
T_{r,21} = -\frac{2\pi \xi}{K \beta} \left[ \beta (\varpi - \eta_1) \cos[(\varpi - \eta_2) \beta] + (\varpi \eta_2 - \eta_2 \beta^2 + 1) \sin[(\varpi - \eta_2) \beta] \right]
\] (14)

\[
T_{r,22} = \frac{\varpi \eta_2}{\eta_2} \cos[(\varpi - \eta_2) \beta] - \frac{1}{\beta \eta_2} \sin[(\varpi - \eta_2) \beta]
\] (15)

(3) Transfer matrix of neck

Transfer matrix of the neck \( T_N \) is written as follows [6]:

\[
T_N = \begin{bmatrix} \cosh \left( \frac{\xi s}{c} \right) & Z_n \sinh \left( \frac{\xi s}{c} \right) \\ \frac{1}{Z_n} \sinh \left( \frac{\xi s}{c} \right) & \cosh \left( \frac{\xi s}{c} \right) \end{bmatrix}
\] (16)

where, \( \xi \) is the complex coefficient for unsteady viscous friction effect.

\[
\xi = 1 + \sqrt{\frac{4\nu}{d_s^2} + \frac{4\nu}{d_s^2}}
\] (17)

In Eq. (16), the characteristic impedance of the neck \( Z_n \) is calculated by putting \( d_s = d \) in Eq. (2). The transfer matrix \( T_S \) of the silencer is described by the dot product of the transfer matrices \( T_N, T_V \) on the neck and vessel, if the existence of the transmission area between the neck and vessel may be neglected. The transmission loss of the silencer with the hemispherical vessel is finally calculated from the Eq. (1) stepping these procedures.

Lumped Parameter Model

In the lumped parameter model, the resonance frequency \( f_r \) and the transmission loss \( TL_r \) at the resonance frequency are easily obtained from the dimensions of the provided Helmholtz silencer and fluid properties. Putting \( \varpi = d^2 \sqrt{\nu} \) [6],

\[
f_r = \frac{\sqrt{3}}{2\pi} \sqrt{\frac{K}{\rho D^{1/2}}}
\] (18)

\[
TL_r = 20 \log_{10} \left( 1 + \frac{c}{64\nu d_s^2} \right)
\] (19)

In Eq. (18), it is effective to consider the two dimensional indexes on the neck and vessel. One of them is \( \varpi = d^2 / l \) which implies the ratio of neck area and length. The other one is vessel diameter \( D \) which relates to the vessel volume.

EXPERIMENTAL APPARATUS

Figure 2 shows the schematic diagram of the experimental apparatus. Hydraulic fluid from a variable displacement vane pump is delivered to the test silencer through a main line with a diameter of \( d_s = 21 \text{ mm} \). The experiment was conducted under the condition of a supply mean pressure of \( P_s = 7 \text{ MPa} \) and a flow rate of \( Q = 25-30 \text{ L/min} \). The fundamental frequency of source flow ripple could be arbitrarily changed by the pump rotational speed by using a motor inverter. The measurement of the transmission loss was achieved by means of the 4-pressures and 2-systems method [4]. It was performed by utilizing the four pressure transducers and two valves which were located upstream and downstream of the test silencer as shown in Fig. 3. The steady flow rate was monitored by a flow meter, and the oil temperature in a hydraulic tank was controlled by an oil cooler which kept it constant at 40 ± 1 °C during the...
experiment. Electric signals from pressure transducers were amplified and converted into the frequency domain, and these data were finally analyzed by a computer. Table 1 shows the main parameters of the experimental system. Three kinds of hemispherical vessel were manufactured by a 3D machine tool having numerical control as the diameters are $D = 80$ mm, 115 mm, and 170 mm. The diameter $d$ and length $l$ of the necks are also listed in Table 2. As may be seen, the difference between No.1, No.2 and No.3 is the length dimension, which was chosen arbitrarily. On the other hand, the neck dimensions of the No.1, No.4 and No.5 were determined so that the value of $\varepsilon$ remained equal. This is why the resonance frequency $f_r$ is the function of $\varepsilon$ as written in Eq. (18).

In the calculation, the neck length $l_n$ was used in consideration of the open end correction as follows [7]:

$$l_n = l + \frac{8}{3\pi}d$$  \hspace{1cm} (20)

Table 1 Specification of parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed of sound $c$ [m/s]</td>
<td>1380</td>
</tr>
<tr>
<td>Density of oil $\rho$ [kg/m$^3$]</td>
<td>874</td>
</tr>
<tr>
<td>Kinematic viscosity of oil $\nu$ [m$^2$/s]</td>
<td>3.2×10$^{-5}$</td>
</tr>
<tr>
<td>Bulk modulus of oil $K$ [GPa]</td>
<td>1.66</td>
</tr>
</tbody>
</table>

Table 2 Designed dimensions of neck

<table>
<thead>
<tr>
<th>Neck</th>
<th>$d$ [mm]</th>
<th>$l$ [mm]</th>
<th>$\varepsilon$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.1</td>
<td>12</td>
<td>45.0</td>
<td>3.20</td>
</tr>
<tr>
<td>No.2</td>
<td>14</td>
<td>101</td>
<td>1.42</td>
</tr>
<tr>
<td>No.3</td>
<td>16</td>
<td>220</td>
<td>0.650</td>
</tr>
<tr>
<td>No.4</td>
<td>14</td>
<td>62.9</td>
<td>3.20</td>
</tr>
<tr>
<td>No.5</td>
<td>16</td>
<td>84.3</td>
<td>3.20</td>
</tr>
</tbody>
</table>

CONSIDERATION ON ATTENUATION CHARACTERISTICS

The attenuation characteristics are generally presented by the transmission loss in the frequency domain. Figure 4 shows a typical comparative example of experimental and calculated results of the transmission loss $TL$. In order to evaluate the novel distributed parameter model proposed in this study, the results from the lumped parameter model are also illustrated in the same diagrams. It is found that the distribution around the resonance frequencies is nearly equal in the theoretical calculations and experimental results. However, the experimental transmission loss at resonance frequency are lower than that of the mathematical models as reported in the previous papers [8]. Throughout this study, it should be noted that experimental results of $TL_r$ are roughly half lower than the distributed parameter model estimates, and a third lower than the lumped parameter model estimates.

First, in order to examine the influence of the vessel diameter $D$ on transmission losses, the experimental results of the resonance frequency $f_r$ and the transmission

Fig. 4 Experimental and calculated results of the transmission loss $TL$ ($D=115$ mm, Neck No.2)
loss at resonance frequency $TL_r$ are compared to the proposed distributed parameter model as shown in Fig. 5. This comparison is based on necks No. 1 to No. 3. It is evident from Fig. 5(a) that resonance frequency $f_r$ is reduced to be inversely proportional to the $2/3$ power of the vessel diameter $D$. And then the results of the proposed model agree well with the experimental values within the range of the diameter from $D=80$ mm to 170 mm. It can be seen from Fig. 5(b) that the calculated and experimental results are almost constant. This is because the vessel diameter $D$ is not fully related with the value of the transmission loss at resonance frequency as described in Eq. (19).

Next, the influence of the neck dimensions is investigated. Figure 6 shows the relationship between the value $\varepsilon$ and the attenuation characteristics. The diameter of the volume is $D=80$ mm. As seen from this figure and Eq. (18), the resonance frequency is proportional to the square root of the index $\varepsilon$. Experimental values are in good agreement with the theoretical calculation of the two models.

By appropriately designing the combination of the neck length and neck diameter, the resonance frequency may be kept constant based on the lumped parameter model shown in Eq. (18). In this study, the value of $\varepsilon=3.20$ mm was selected as seen in the neck No.1, No.4 and No.5 in Table 2. As a result, the designed resonance frequencies $f_r$ were calculated to be 300 Hz, 540 Hz and 935 Hz corresponding to the vessel diameters $D=170$ mm, 115 mm and 80 mm respectively. Figure 7 shows the results obtained from the distributed parameter model and experimental data, taking the neck diameter $d$ on the horizontal axis. From this figure, it is clear that the resonance frequencies are almost constant when the vessel diameter is identical, and the experimental results are compared well with the theoretical results. It is shown that the attenuation characteristics are not affected by the size of the neck diameter, even if the neck diameter $d$ (i.e. twice of inlet radius $r_1$ of the vessel) are varied from 12 mm to 16 mm experimentally. As a result, the validity of novel proposed model has been revealed within the range of the vessel radius ratio $r_1/r_2$ equals 0.05.

**CONCLUSION**

This paper presented a novel distributed parameter model for a Helmholz silencer with hemispherical vessel shape. The model was constructed considering one
dimensional spherical wave propagation for radial direction. In order to investigate the effectiveness of the proposed distributed parameter model, theoretical calculations are compared with the experimental results in the view of the attenuation characteristics. As a result, it was confirmed that the predicted values of the proposed model agreed well with not only the experimental values but also the lumped parameter model on the frequency domain. In addition, the suitability of the proposed theory was also verified in the case of the vessel and neck with various dimensional specifications.

REFERENCES


