Proceedings of the 8th JFPS International
Symposium on Fluid Power, OKINAWA 2011
Oct. 25-28, 2011

1B1-1

AN INDIRECT MEASUREMENT METHOD OF TRANSIENT PRESSURE AND FLOW RATE IN A PIPE USING STEADY STATE KALMAN FILTER

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ABSTRACT

The purpose of this study is to estimate fluid transients in a pipe using measured pressure values of three points. In this study, method of estimating fluid transients by combining a steady-state Kalman filter and an optimized finite element model of pipeline dynamics was proposed. In this paper, fluid transients in a pipe were estimated off-line by using experimental values in order to show basic performance of the Kalman filter. For comparison, the method of characteristics and was applied to the same data. Estimation results of the Kalman filter show good agreement with the result of the method of characteristics. In addition, Steady flow rate estimated by the Kalman filter was compared with the flow rate measured by flow meters. In this comparison, it was confirmed that the estimated result is proportional to the measurement result.

KEY WORDS

Kalman Filter, Pipe Flow, Optimized Finite Element Model, Simulation

NOMENCLATURE

A : Cross section of the pipe line
$\boldsymbol{A_p}$, $\boldsymbol{B_p}$: coefficient matrix of OFEM model
A, B, E, F : coefficient matrix
c : wave speed
$\boldsymbol{C}$: output matrix
G : coefficient matrix
K_T : Kalman Filter gain
P : Solution of Riccati eqation
$\boldsymbol{p}$: pressure vector
$\bar{\boldsymbol{p}}$: input vector of OFME model
p_f : a function of flowrate
$\boldsymbol{Q}$: covariance vector of system noise
$\boldsymbol{q}$: flow rate vector
R : covariance of sensor noise
t : time
v : sensor noise
w : system noise
x : state variable of OFEM model
ρ : Density

Subscript
N : number of grid point
up : upstream end
mid : mid point
$down$: downstream end

INTRODUCTION

Two major methods of understanding fluid transients are measurements and simulations. However, pressure and flow rate of a point which is not a measurement point cannot be obtained by the measurement with a sensor. Moreover, the simulation that uses the approximate expression includes error margin. Recently, the study on the technique for combining two techniques, the measurement and the simulation, is done. For example, in measurement of the flow rate in a pipe, remote measurement of instantaneous flow rate was proposed by Yokota (1991)[1)] and a real time measuring method of unsteady flow rate and velocity employing a differential pressure in a pipe was proposed by Zhao(1986)[2)]. In these techniques, the flow rate of one point was obtained from the measurement of the pressure of two points by using the hydraulic pipeline dynamics.

On the other hand, Measurement-Integrated Simulation was proposed by Funamoto and Hayase (2004)[3)] .In these techniques, a lot of calculations are required, because fundamental equations of three dimensional or two dimensional flow are solved.

In this study, method of estimating fluid transients by combining a steady-state Kalman filter and an optimized finite element model (OFEM model) of pipeline dynamics was proposed. In this estimation, it is expected that not only flow rate of various points but also pressure of various points can be measured indirectly from measured pressure of three points without accurate setting of initial condition.

OUTLINE OF INDIRECT MEASUREMENT METHOD

OFEM model is based on the equation of motion of fluid flow in a circular pipe and the continuity equation. Based on an interlacing grid system as shown in Fig.1, assuming one-dimensional flow, and neglecting a convection term, the equation of motion of fluid flow in a circular pipe is written as follows [4)]

$$\frac{\mathrm{d}\boldsymbol{q}}{\mathrm{d}t}+\frac{A}{\rho}\boldsymbol{Bq}+\frac{A}{\rho}\boldsymbol{F}\bar{\boldsymbol{p}}+\boldsymbol{p_f}=0 \qquad (1)$$

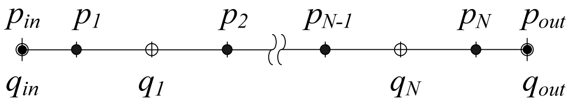

Fig.1 Interlacing grid system

The continuity equation is written as follows.

$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t}+\frac{\rho c^2}{A}\boldsymbol{Eq}=0 \qquad (2)$$

Where, the matrices ***B***, ***E*** and ***F*** are calculated from the finite element approximation. Elements of the vector ***p*** and ***q*** are flow rate variable and pressure variable at the grid points

$$\boldsymbol{q}=\left[q_{up},q_1,\dots q_{N-1},q_{down}\right]^T \qquad (3)$$

$$\boldsymbol{p}=\left[p_1,p_2\dots p_N\right]^T \qquad (4)$$

$\bar{\boldsymbol{p}}$ is pressure input of OFEM model and $\boldsymbol{p_f}$ is a function of flowrate at grid point.

$$\bar{\boldsymbol{p}}=\left[p_{up},p_{down}\right]^T \qquad (5)$$

$$\boldsymbol{p_f}=\left[p_f\left(q_{up}\right),p_f\left(q_1\right)\dots p_f\left(q_{N-1}\right),p_f\left(q_{down}\right)\right]^T \qquad (6)$$

OFEM model can be written as follows by a state space equation[4)]

$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t}=\boldsymbol{A_p x}+\boldsymbol{B_p}\bar{\boldsymbol{p}} \qquad (7)$$

Where, the state variable vector is

$$\boldsymbol{x}=\left[\boldsymbol{q}^T,\boldsymbol{p}^T\right]^T \qquad (8)$$

When the measurement noise is considered, OFEM model can be written as follows by a state space equation[4)]

$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t}=\boldsymbol{A_p x}+\boldsymbol{B_p}\bar{\boldsymbol{p}}+\boldsymbol{Gw} \qquad (9)$$

$$p_{mid}=\boldsymbol{Cx}+v \qquad (10)$$

Filer equation of Kalman Filter is written as follows.

$$\hat{\mathbf{x}}[n]=\boldsymbol{A_p}\left(\boldsymbol{I}-\boldsymbol{K_T C}\right)\hat{\mathbf{x}}[n-1]+\boldsymbol{A_p K_T}p_{mid}+\boldsymbol{B_p}\bar{\boldsymbol{p}} \qquad (11)$$

Where, $\boldsymbol{K_T}$ is Kalman Filter gain, ***P*** is solution of Riccati eqation.

$$\boldsymbol{K_T}=\boldsymbol{PC}^T\left(R+\boldsymbol{CPC}^T\right)^{-1} \qquad (12)$$

$$\boldsymbol{P}=\boldsymbol{A}(\boldsymbol{P}-\boldsymbol{PC}^T[\boldsymbol{CPC}^T+R]^{-1}\boldsymbol{CP})\boldsymbol{A}^T+\boldsymbol{GQG}^T \qquad (13)$$

Where, $\boldsymbol{Q}$ is covariance vector of system noise, R is covariance of sensor noise

$$\boldsymbol{Q} = E(\boldsymbol{w}[n]\boldsymbol{w}[n]^T) \tag{14}$$

$$R = E(v[n]^2) \tag{15}$$

A block diagram for estimation is illustrated in Fig.1. Three sensors are attached at a pipe to pick up transient pressure at the upstream-end p_{up}, the mid-point p_{mid}, and the downstream-end p_{down}. In this study, sensor noise of p_{mid} is considered as measurement noise. The steady state Kalman Filter is used to estimate the fluid transition. In OFEM model, after the observation time passes enough, the Kalman filter is settled to the filter of a fixed coefficient. Therefore, it only has to request the steady state Kalman filter gain, and the Riccati equation need not be solved in real time.

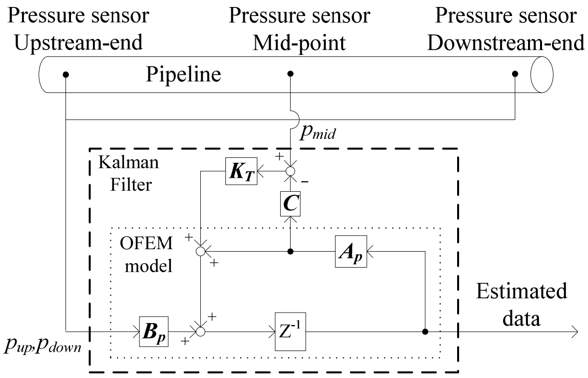

Fig.2 Schematic diagram of a proposed Kalman filter

COMPARISON WITH A METHOD OF CHSRACTERISTICS

Fluid transients in a pipe were estimated off-line by using these experimental values in order to show the basic performance of the Kalman filter. Fig.2 shows experiment instrument. Test parameters are listed in Table 1. Working fluid was tap water of 16 degree Celsius. Three pressure sensors were attached at the pipe to pick up the transient pressures at upstream-end, the mid-point, and, the downstream-end. The experimental data is shown in Fig.4. By switching the spool valve, the upstream pressure was increased quickly. Because of pressure wave travelling along the pipe, the mid-point pressure and the downstream-end pressure showed oscillations.

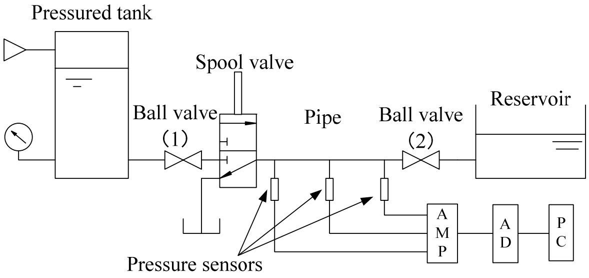

Fig.3 Experimental pipeline and measurement system

Table1 Parameters for experiment and simulation

Wave speed	1310 m/s
Tank pressure	0.2 MPa
Length of pipe	36 m
Number of elements	5
Diameter of pipe	10 mm
Kinematic viscosity	1.05 cSt
Density	999 kg/m^3
Sampling time	2 ms

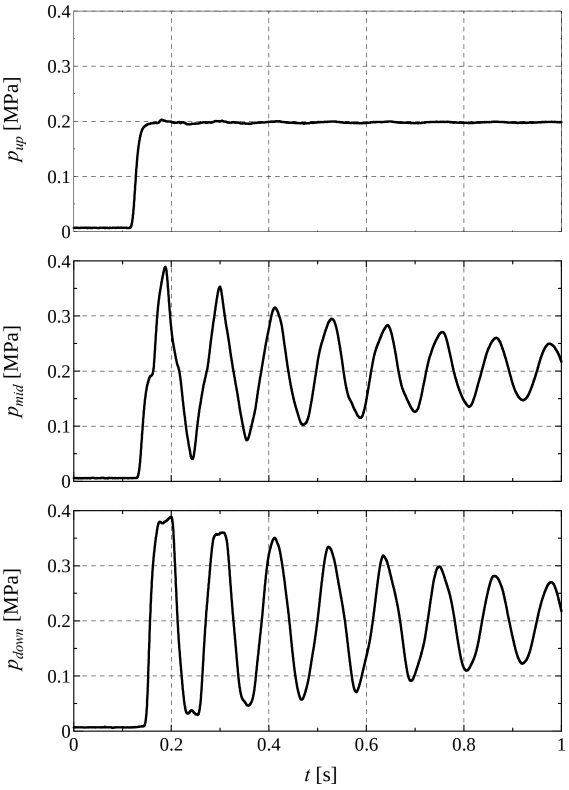

Fig.4 Experimental results of pressures p_{up}, p_{mid} and p_{down} used for the Kalman filter

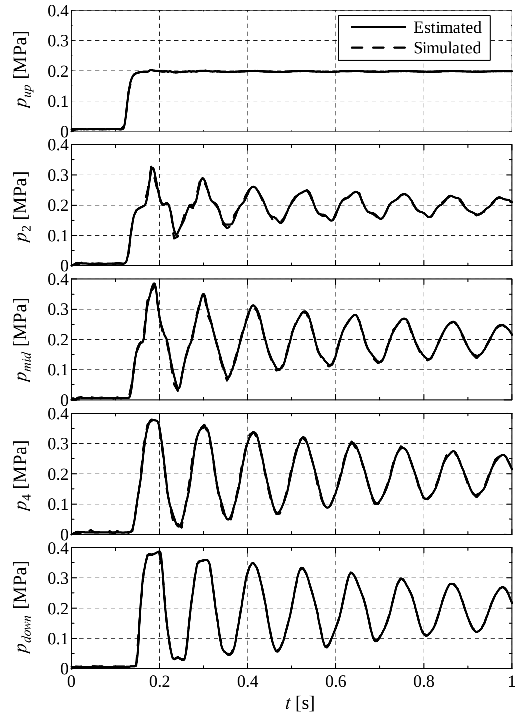

Fig.5 Transient flow rates obtained by the Kalman filter and the method of characteristics

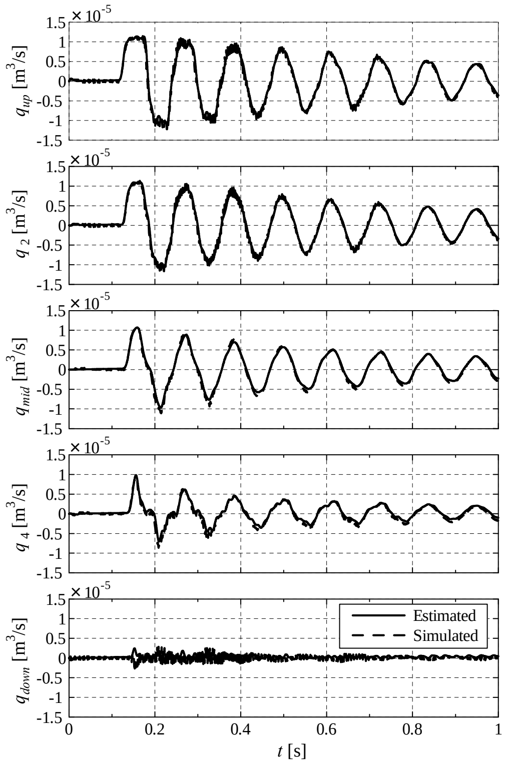

Fig.6 Transient flow rates obtained by the Kalman filter and the method of characteristics

Fig.5 shows the pressure at uniformly spaced grid points in a pipe and Fig.6 shows the flow rate at same points. For comparison, it was also simulated by the method of characteristics using the same data. The estimation results showed good agreement with the simulation results. Therefore, it was confirmed that the time spatial distributions of pressure and flow rate can be precisely estimated by the Kalman Filter.

COMPARISON WITH FLOW RATE OBTAINED BY A FLOW METER

Steady flow rate estimated by the Kalman filter was compared with the flow rate measured by flow meters in order to confirm the static characteristics of the Kalman Filter. Fig.6 shows experiment instrument. Test parameters are listed in Table 2. The pipe (1) was made of a stainless pipe of 20mm in inner-diameter and 2.5mm thickness. The total length of estimated section was 3.2m. Working fluid was hydraulic oil of 30 degree Celsius. The upstream of the estimated section was connected the relief valve (2), pressure gauge (3) and pump unit (4). At first, the throttle valve (5) was closed. Then, the hydraulic fluid is discharged by the pump unit (4) and returns to a tank through a relief valve (3). Pressure was set as 4MPa by the relief valve (3). Secondly, when a throttle valve (5) was opened, the hydraulic oil is discharged from the pump unit (4), and back to tank through the pipe (1), throttle valve (5), and flow meter (6). Then, the pressure falls so that flow rate increases. Three pressure sensors were attached at the pipe to pick up the pressures at upstream-end, the mid-point, and, the downstream-end. In this experiment, distance of each sensor was 1.6m. In addition, a flow meter is attached to the down stream side of the throttle valve. Therefore, estimation value of the steady flow rate can be compared with the measured value.

Table 2 Parameters for experiment and simulation

Wave speed	1310 m/s
Relief pressure	4 MPa
Length of pipe	3.2 m
Number of elements	5
Diameter of pipe	20 mm
Kinematic viscosity	40 cSt
Density	850 kg/m^3
Sampling time	1 ms

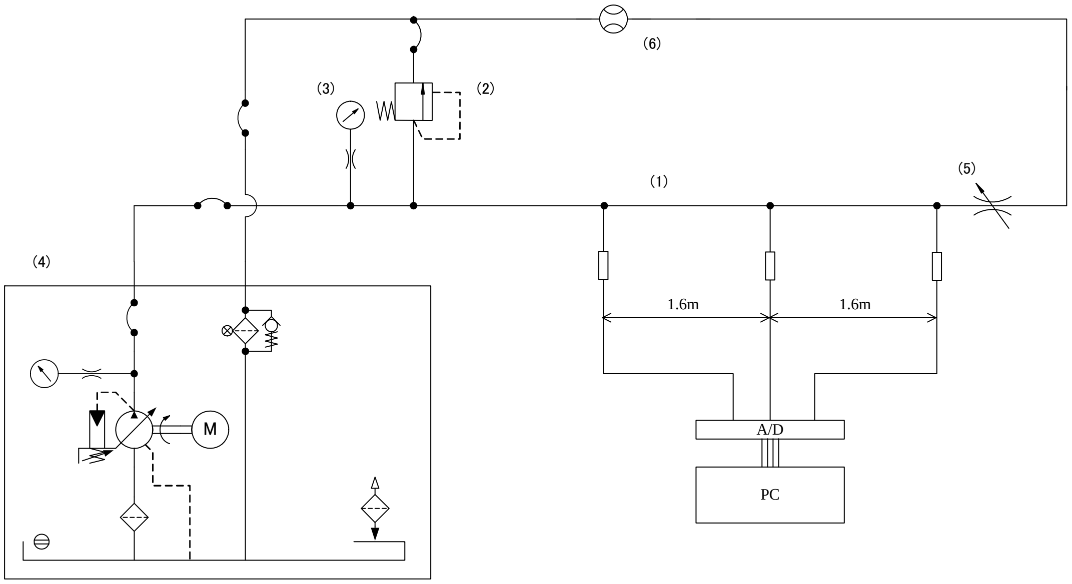

Fig.6 Experimental pipeline and measurement system

Fig.7 shows an example of measured pressure. The pressure at mid-point p_{mid} was shown. Pressure showed oscillations in this figure. In this instrument, noise of amplifier is 0.06MPa. Therefore, it was thought that these oscillations were pressure pulsations caused by a pump unit. Fig.8 shows the flow rate estimated by the Kalman filter, and Fig.9 shows the flow rate obtained by the flow meter. Estimated flow rate shows oscillation. The oscillation of measured flow rate was less than the oscillation of estimated flow rate. It was thought that a cause of the oscillation of estimated flow rate was the pressure pulsation. Therefore, flow rate pulsation was possible to be estimated by Kalman Filter. Estimation results of the Kalman filter show good agreement with the result of measurement.

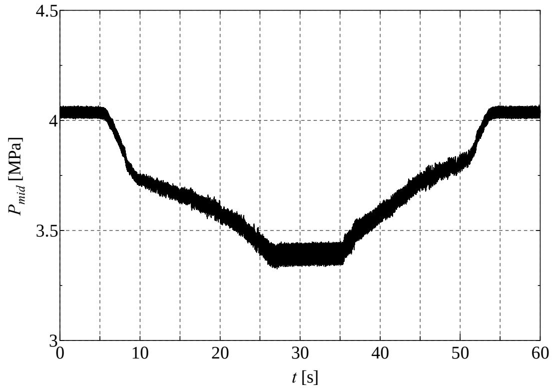

Fig7 Measured pressure at mid-point

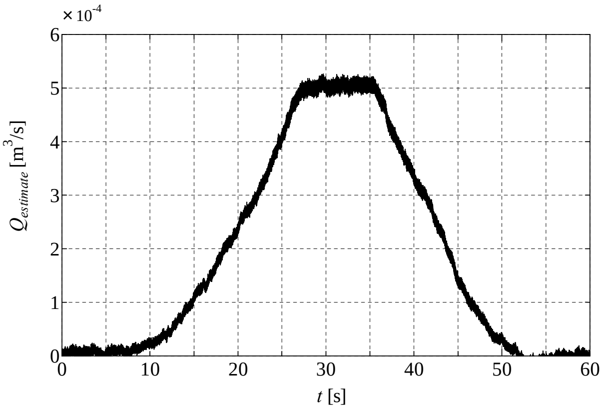

Fig.8 Estimated flow rate at mid point

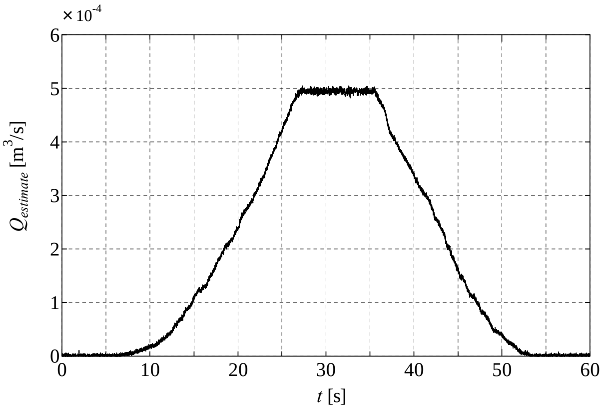

Fig.9 Measured flow rate by flow meter

Fig.10 shows relation between estimated flow rate and measured flow rate. The horizontal axis shows the measured flow rate and the vertical axis shows the estimated flow rate. The oscillation of estimated flow rate influences the results. Moreover, the hysteresis was slightly confirmed. However, it was confirmed that the estimated result is good proportional to the measurement result.

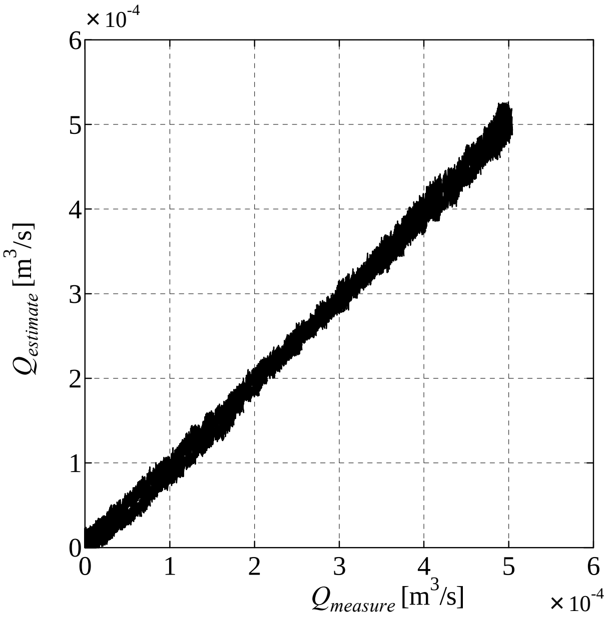

Fig.10 Relation between estimated flow rate and measured flow rate

CONCLUSION

In this paper, method of estimating fluid transients by combining a steady-state Kalman filter and an optimized finite element model of pipeline dynamics is discussed. Pressure and flow rate in a pipe are estimated from measured data of an experiment. As a result, it is confirmed that the estimation results of the Kalman filter show good agreement with the result of the remote measurement method for unsteady flow.

In addition, Steady flow rate estimated by the Kalman filter was compared with the flow rate measured by flow meters. In this comparison, it was confirmed that the estimated result is proportional to the measurement result.

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