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# THERMAL MODEL OF A TANK FOR SIMULATION AND MASS FLOW RATE CHARACTERIZATION PURPOSES

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## ABSTRACT

The paper focuses on the modeling of heat transfer in pneumatic systems. The main aim of this work is to represent these phenomena during the charge or the discharge of a tank for complex circuit simulation or for the identification of instantaneous mass flow rate.
In the first part, a macroscopic model will be proposed. It is based on the dimensional analysis theory. This approach enables the main heat transfer mechanisms to be identified according to flow conditions, pressure levels and tank shape. A relation between the corresponding dimensionless groups is then used to develop a general model for computing the heat exchange coefficient according to the system state and geometry. The identification procedure is presented and the first simulation results show good agreement between experimentation and theory.

## KEY WORDS

Pneumatic systems, orifice modeling, thermal exchange model, identification procedures, experimentation

## NOMENCLATURE

**Physical constants**

$g$ : acceleration of gravity [9.81 $m/s^2$]
$c_v$ : specific heat at constant volume [717 $J/(kg.K)$]
$c_p$ : specific heat at constant pressure [1004 $J/(kgK)$]
$r$ : gas constant [287 $J/(kg.K)$]
$\gamma$ : ratio of specific heat [1.4]

**Physical parameters**

$b$ : critical pressure ratio
$C$ : sonic conductance [$m^3/(Pa.s)$]
$D$ : characteristic diameter [$m$]
$k$ : conductivity [$W/m$]
$m$ : mass [$kg$]
$n$ : polytropic index
$P$ : static pressure [$barA$]
$Q$ : heat [$J$]
$q_m$ : mass flow rate [$kg/s$]
$S$ : surface of heat exchange [$m^2$]
$T$ : static temperature [$K$]
$U$ : internal energy [$J/kg$]
$V$ : volume of the tank [$m^3$]
$\delta_1$, $\delta_2$, $\zeta$, $\zeta'$: heat exchange parameters
$\lambda$ : heat exchange coefficient [$W/m^2/K$]
$\mu$ : viscosity [$kg/(m.s)$]
$\rho$ : density [$kg/m^3$]

**Dimensionless groups**

$Nu$ : Nusselt number
$Gr$ : Grashof number
$Pr$ : Prandtl number
$Ra$ : Rayleigh number
$Re$ : Reynolds number

**Exponents and indices**

*d* : downstream
ext : environnement
*in* : inlet
*out* : outlet
*ref* : reference value
*u* : upstream
*V* : tank
*W* : wall of the tank

## INTRODUCTION

In pneumatic systems, temperature is a critical variable when modeling or characterizing pneumatic systems. It influences not only the fluid properties but also the system performances. It can be an important value to predict the dynamic behavior of a system or to determine the sizing of components such as compressor, cylinder chambers, tanks, etc. However it is still difficult to directly measure it in transient conditions. Indeed, thermocouples do not enable measurements at high frequencies due to their high time constant. Moreover, its measurement is punctual while a macroscopic model focuses rather on the equivalent homogeneous temperature than on its spatial distribution. This leads to many difficulties at the modeling stage and at the model validation or experimental phases. The study of an accurate macroscopic thermal model for a tank constitutes the main purpose of the proposed paper.
There are here two modeling objectives. The first one aims at implementing a model, which can be used to identified the mass flow rate characteristic according to the new proposal for characterizing orifices in transient conditions [1], but using standard tanks [2]. The second one is the simulation of complex circuits such as braking systems for railway or heavy vehicles, for which the sizing of energy storages (tanks) are depending on official safety requirements. The presented work investigates the discharge of a tank, and tries to analyze the heat transfer problem. As the success of our first objective is conditioning the applicability of the development to simulation of complex pneumatic circuits, the paper focuses mainly on the modelling of the temperature transient in a conventional tank during charge or discharge (Fig 1). The dynamic behaviour during the discharge process is then given by (3).

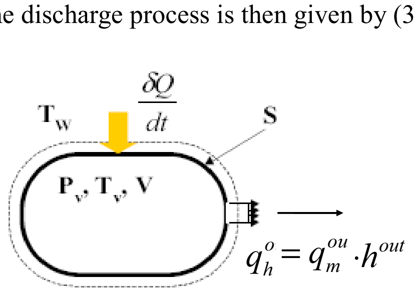


Figure 1: Charge or discharge process in a tank.

During the last decade, a method for characterizing the instantaneous mass flow rate of pneumatic components was developed by the Tokyo Institute of Technology and is based on the charge or discharge of a tank. In 1996, Kagawa and Kawashima [3] used an "isothermal" tank to identify the mass flow rate through an orifice, assuming the temperature variations are small and the mass flow the same anywhere in the circuit [4]. This method is the basis of the Japanese Standard (JISB8390) [5]. Assuming an isothermal process, the mass flow rate is directly given by the measurement of the pressure and its derivative according to (1).

$$q_{m_{out}} = -\frac{V}{r\gamma T_v}\frac{dP_v}{dt} \tag{1}$$

This interesting approach was then extended first to identify the mass flow rate characteristic of a servovalve in the frequency domain [6], second to introduce correction terms considering the influence of the circuit between the tank and the component to be characterized [7], and third to analyze the influence of a temperature correction term [8]. In fact, for an "isothermal" tank, the process cannot be exactly isothermal if the pressure gradient is over a certain value (about 1 $bar/s$). Due to this condition, the isothermal tank still shows some limitations but when the size is well-adapted to the circuit, it can efficiently be used for component characterization and as a mass flow generator [1].
These works show also that for conventional tanks the modelling of the heat exchange is essential in order to reach a good precision when identifying the mass flow rate characteristic of a component [9]. Polytropic models (2) and models with a constant heat exchange coefficient (4) do not enable the required precision to be reached.

$$q_{m_{out}} = -\frac{V}{rn(t)T_v(t)}\frac{dP_v}{dt} \tag{2}$$

Up to now, only a few works have tried to use conventional tanks for identifying mass flow characteristics. For example in 1989, assuming that the process is isentropic for very fast discharge, Wencan and de Las Hers proposed a first method [10, 11] but these approaches are only usable in sonic conditions. Benchabane was the first in 1994 to develop such a method [12]. His work constitutes the basis of a French standard NF E49-300 [13]. In this case, the polytropic index $n$ is adjusted according to the temperature measured in the tank at any time. There were here important limitations due to the size of the tank in order to reach slow transient conditions and the need to equip the tank with a fan air for temperature homogeneity. In 2004, Kuroshita [14] defined a method still using the temperature measurement in the tank but enabling to adjust the polytropic index from the initial and final

values of the temperature.
However, all these approaches show some drawbacks such as the validation of the assumptions and their sensitivity to flow conditions. Figure 2 shows clearly the difficulty. It plots the instantaneous polytropic index and the heat exchange coefficient according to time during the discharge of a tank through the same circuit but starting from different initial pressures. Therefore it justifies the development presented here that uses a physical approach to properly model the heat exchange phenomena.

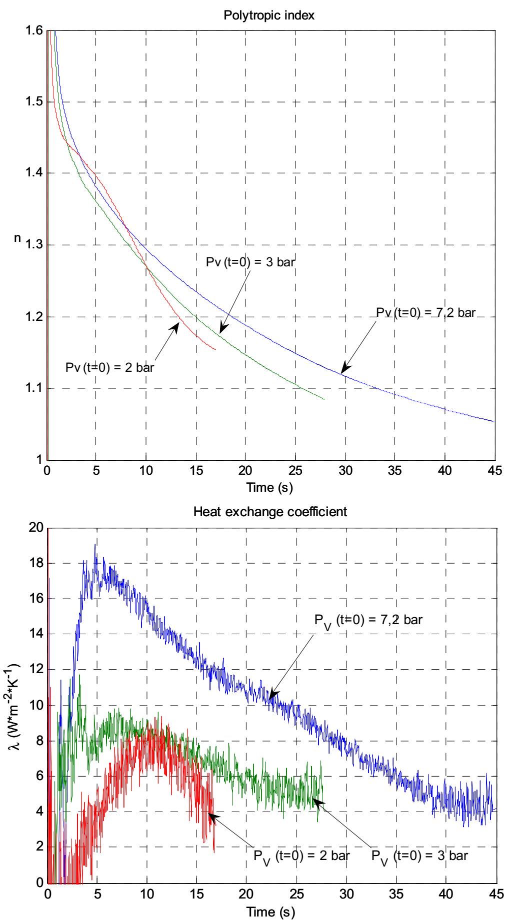


Figure 2: Polytropic index and heat exchange coefficient for different initial pressures in a discharge process.

In the first part, a macroscopic model will be proposed. It is based on the dimensional analysis theory. This approach enables the main heat transfer mechanisms to be identified according to flow conditions, pressure levels and tank shape. A relation between the corresponding dimensionless groups is then used to develop a general model for computing the heat exchange coefficient according to the system state and geometry. The identification procedure is presented and the first simulation results are compared with experimentation.

## THERMAL EXCHANGE MODELING

Considering the discharge of a tank (Fig.1), if the chamber volume is large enough, the kinetic energy of the fluid in the chamber can be neglected. The mass conservation law and the energy conservation law enable the complete description of the dynamic behaviour of the gas in the chamber. Considering the heat exchanged with the environment, without any mechanical work, the first law of the thermodynamics can be applied to this opened system. With the hypothesis of a perfect gas, and assuming that, at any time, the pressure $P_V$, the temperature $T_V$ and the density $\rho_V$ of the gas are uniform in the chamber and equal to their mean value according to space, the state model of the system can be described by (3) using pressure and temperature in the volume as state variables.

$$\begin{cases} \dfrac{dP_V}{dt} = -\dfrac{r\gamma T_V}{V} q_{m_{out}} + \dfrac{\gamma-1}{V} \cdot \left(\dfrac{\delta Q}{dt}\right)_V \\ \dfrac{dT_V}{dt} = \dfrac{(\gamma-1)T_V}{P_V V}\left[-rT_V q_{m_{out}} + \left(\dfrac{\delta Q}{dt}\right)_V\right] \end{cases} \quad (3)$$

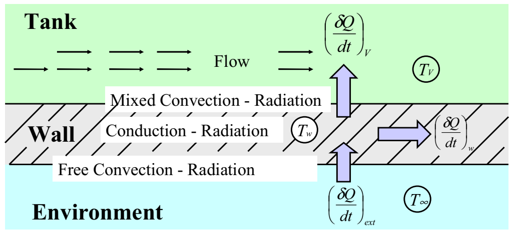


Figure 3: Thermal exchange between tank and gas.

Generally, 3 types of heat transfer are considered [15]:
- ✓ radiation: the transfer of thermal energy is due to absorption or emission of electromagnetic radiation (it is the only mechanism without a material medium);
- ✓ conduction: the heat transfer is due to molecular movement inside the medium;
- ✓ convection: the heat transfer occurs between a solid and a fluid in association with mass transfer.

It is thus necessary to study more precisely the heat exchanges taking place between the gas in the tank and the environment (Fig.3). Classical hypotheses used in pneumatic chambers rely on the assumption that the thermal conductivity and the heat capacity of the wall material are sufficiently large compared with those of air, the wall temperature is therefore considered as a constant and the heat exchanged (4) can be described by a convection heat transfer model expressed by the Newton's Law using a convection coefficient $\lambda$.

$$\left(\frac{\delta Q}{dt}\right)_V = \lambda S\left(T_V - T_W\right) \quad (4)$$

Benchabane [12] and Det [16] have tried different approaches to obtain an evaluation of the heat exchange coefficient $\lambda$ according to flow conditions. For example, Det used the Eichelberg's model (5) [17], but this improves only slightly the precision and requires a calibration procedure for each circuit.

$$\lambda = \lambda^{ref}\sqrt{\frac{P_V T_V}{P_V^{ref} T_V^{ref}}} \tag{5}$$

## CONVECTION PHENOMENA

The convection can be split in two phenomena according to the phenomena influencing mass transfer:

- ✓ natural convection: it occurs when the mass transfer is due to a temperature gradient;
- ✓ forced convection: the mass transfer is here imposed by a difference of pressure.

The dimensional analysis is used at the mascroscopic scale for modelling physical phenomena depending from several variables. For convection, it is shown that 3 dimensionless groups are required: Nusselt (*Nu*), Prandtl (*Pr*) and Grashof (*Gr*) numbers. According to the Buckingham's theorem [18], the relation between these groups is then given by (6):

$$Nu = \zeta Gr^{\delta_1} Pr^{\delta_2} \tag{6}$$

The parameters $\zeta, \delta_1$ and $\delta_2$ are constants that can be obtained experimentally by varying characteristic values of the dimensionless groups defined as follows (7), (8), and (9):

$$Nu = \frac{\lambda D}{k} \tag{7}$$

$$Gr = g\frac{\left(T_W - T_V\right) D^3 P_V^2}{\mu^2 r^2 T_V^3} \tag{8}$$

$$Pr = \frac{c_p \mu}{k} \tag{9}$$

$$r_d = Gr / \left(Re\right)^2 \tag{10}$$

$$R_a = GrPr \tag{11}$$

The main advantage of this approach relies on the physical interpretation of the phenomena that are associated to the dimensionless groups [15]:

- ✓ the Nusselt number corresponds to the ratio between the heat power exchanged by convection and conduction;
- ✓ the Prandtl number characterizes the velocity distribution versus the temperature distribution;
- ✓ the Grashof number is the ratio between the product of Archimedes and inertial force, and viscous forces.

Combining these dimensionless groups, the main heat exchange mechanism can be determined:

- ✓ the ratio $r_d$ (10) is used to determine the type of convection phenomena:
  - if $r_d$ «1, natural convection can be neglected,
  - if $r_d$ » 1, forced convection can be neglected,
  - if $r_d \approx 1$, both phenomena have about the same magnitude, it is called mixed convection.
- ✓ the Rayleigh number (eq.11) is also used for natural convection modelling in order to determine the transition between laminar ($Ra < 10^6$) and turbulent convection ($Ra > 10^{10}$).

In the case of the mass flow rate characterization [2] of a component by discharge of a tank (Fig.4), the heat exchange is essentially due to laminar natural convection because the flow velocity is low inside the tank (small *Re* number). Note that mixed convection may occur in specific conditions such as high initial pressure, high component sonic conductance, or small tank volume.

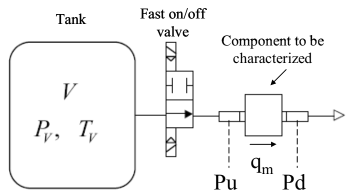


Figure 4: Experimental bench for mass flow rate characterization using tank discharge.

## IDENTIFICATION OF CONVECTION COEFFICIENTS

The experimental bench (Fig.4) is used here for characterizing the heat exchange phenomena. The discharge of the tank is realized through a component with a known mass flow rate characteristic obtained from a direct mass flow rate measurement according to ISO6358:89 [19]. According to (12), deduced from (3), measuring the pressure and the temperature in the tank during the discharge allows the computation of the heat transfer. The instantaneous heat exchange coefficient $\lambda$ is then computed from (13). Median filtering and under sampling is applied to pressure measurement before differentiation, and the temperature is obtain from partial discharges of the same circuit according to the stop method introduced by Kawashima [4].

$$\left(\frac{\delta Q}{dt}\right)_V = \frac{V}{\gamma - 1}\frac{dP_V}{dt} + c_p T_V q_{m_{out}}\left(P_V, T_V\right) \tag{12}$$

$$\lambda = \frac{\left(\frac{\delta Q}{dt}\right)_V}{S\left(T_V - T_W\right)} \tag{13}$$

The experimental results presented in the paper correspond to the discharge of a *45 ℓ* tank through a component with a critical pressure ratio $b = 0.41$ and a sonic conductance $C = 3.39\,10^{-8}\,kg/(Pa.s)$.

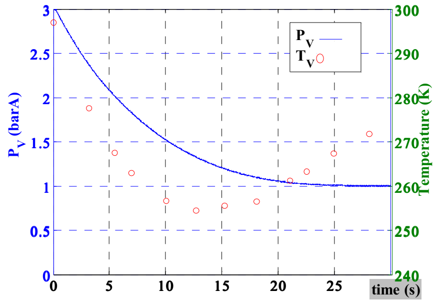


Figure 5: Pressure and temperature during the tank discharge.

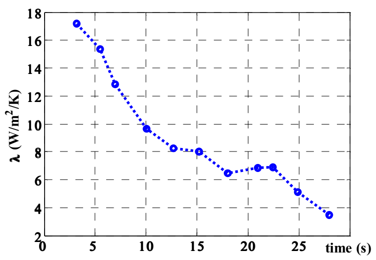


Figure 6: Heat exchange coefficient during tank discharge.

From the experimental data (Fig.5), the dimensionless groups (*Nu*, *Gr*, *Pr*, and *Ra*) can now be computed. Because the air viscosity shows low variations in the range of temperature observed in this kind of application, the Prandtl number can be considered as constant ($Pr \approx 0.715$) and according to (14) deduced from (6), the parameters $\delta_1$ and $\zeta'$ can be identified.

$$\ln(Nu) = \ln(\zeta') + \delta_1 \ln(Gr) \tag{14}$$

with $\zeta' = \zeta Pr^{\delta_2}$

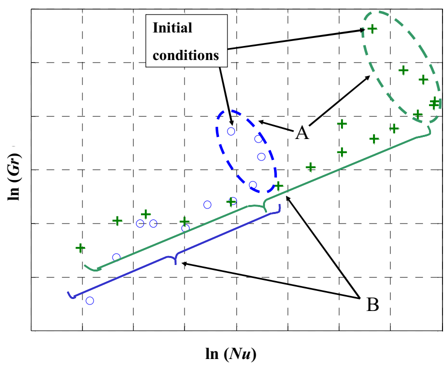


Figure 7: Nusselt number according to Grashof number.

Figure 7 presents the results obtained for several initial pressures. They show clearly two domains. When the discharge starts, complex transitory phenomena occur for a few seconds (A), and the proposed approach can not be applied. However in zone (B), the results are in good agreement with (14) and this part is used to identify the parameters $\delta_1$ and $\zeta'$ of relation (6).

## VALIDATION AND MODEL ANALYSIS

The simulation model of the tank is consequently given by (15):

$$\begin{cases} \dfrac{dP_V}{dt} = -\dfrac{r\gamma T_V}{V} q_{m_{out}} + \dfrac{\gamma-1}{V} \cdot \dfrac{k}{D} \zeta' Gr^{\delta_1} S (T_V - T_W) \\ \dfrac{dT_V}{dt} = \dfrac{(\gamma-1) T_V}{P_V V} \left[ -r T_V q_{m_{out}} + \dfrac{k}{D} \zeta' Gr^{\delta_1} S (T_V - T_W) \right] \end{cases} \tag{15}$$

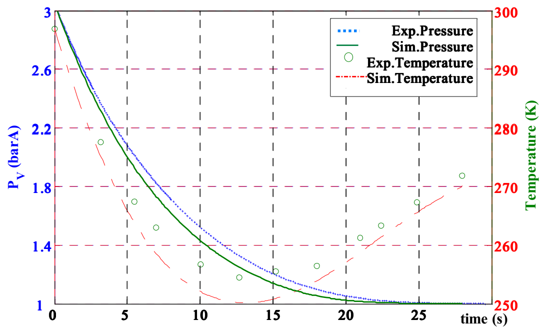


Figure 8: Simulation vs. experimentation when the initial transitory effect is neglected.

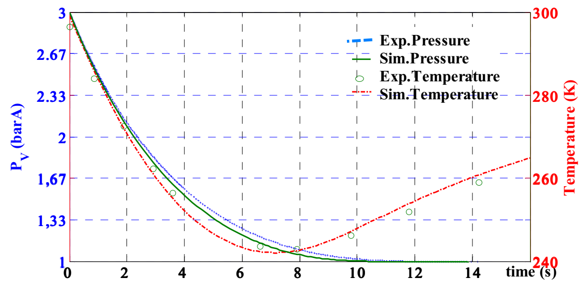


Figure 9: Simulation vs. experimentation when the component is changed.

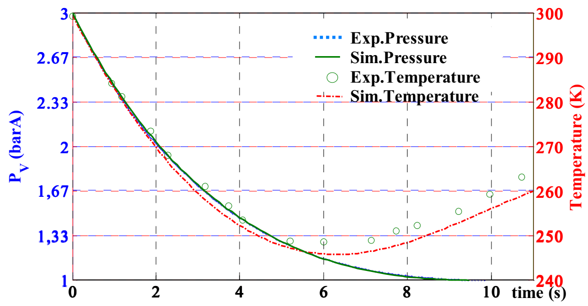


Figure 10: Simulation vs. experimentation when the tank volume is changed.

The simulation model is firstly validated for the same circuit as previously using the heat exchange parameters obtained from zone (B). Figure 8 shows a good agreement with experimental results. The modeling error resulting from the approximation of the heat exchange parameter when the discharge starts, has a low influence on the representation of the temperature and pressure change during the whole discharge.
Finally, if circuit (component) or tank is changed, the results (Fig.9 and 10) from the simulation model are still in good agreement with experimental data.

## CONCLUSION

This paper has proposed a physical approach for modeling heat transfer phenomena during the discharge of a tank. The model is based on dimensional analysis. This approach allows firstly the main thermal effect to be characterized and the heat exchange parameters to be identified from experimentation.
However the simulation results presented here are only given for a single tank, the modeling approach has been extended to other tanks using a shape factor $\xi$ that modify the effective heat exchange area $S$ in order to take into account the tank shape changes (for example the ratio between tank length and diameter). Moreover a forced convection model has been now developed to reach a better approximation of the global convection phenomena (mixed convection) that are observed for high flow rate and pressure level.
One of the main advantages of this modeling approach is that it allows a proper evaluation of the heat exchange coefficient according to different factors such as pressure, temperature, flow conditions and tank shape.
Further works rely now on a rigorous model validation in order to verify the shape factor $\xi$ and to evaluate the influence of other parameters such as tank material (conduction) and painting (radiation).

## REFERENCES


1. K. Kawashima, T. Fujita, and T. Kagawa. Flow rate measurement of compressible fluid using pressure change in chamber. Trans. of the Soc. of Instrument and Control Engineers, E-1(1):252–258, 2001.
2. R. De Giorgi, E. Bideaux, and S. Sesmat. Using inverse model for determining orifice mass flow rate characteristics. 6th Int. Symp. on Fluid Power, pp. 380–385. Tsukuba, Japan, Nov.7-10 2005.
3. K. Kawashima, T. Fujita, T. Kagawa, and J. Jang. Characteristics measurement of pneumatic elements using isothermal chamber. 3rd Int. Symp. on Fluid Power, pp. 253–258. Yokohama, Japan, Nov.4-6 1996.
4. T. Kagawa, K. Kawashima, and T. Fujita. Effective area measurement method using isothermal chamber. Hydraulics and Pneumatics, Vol. 26(1):76–78, 1995.
5. JIS B 8390. Pneumatic Fluid Power Components using Compressible Fluids - Determination of Flow-rate Characteristics, 2000.
6. K. Kawashima, T. Fujuta, T. Kagawa, and J. Jang. Characteristic measurement of pneumatic flow control valves using isothermal chamber. 10th Int. Fluid Power Workshop, pp. 10–15. Bath, UK, Sept. 10-12 1997.
7. B. Han, T. Fujita, T. Kagawa, K. Kawashima, and M. Cai. Flow rate coefficient measurement by using pressure discharge velocity of pneumatic RC circuit. 4th JHPS Int. Symp. on Fluid Power, pp. 143–148. Tokyo, Japan, Nov. 15-17 1999.
8. T Kagawa, T. Wang, Y. Ishii, Y. Terashima, T. Morozumi, T. Mogami, and N. Oneyama. Determination of flow rate characteristics of small pneumatic valves using isothermal chamber by pressure reponse. 7th Symp. on Fluid Control, Measurement and Visualization. Sorrento, Italy, Aug. 25-28, 2003.
9. G. Peng, X. Chai, and W. Fan. A new measurement method of the flow rate characteristics of the regulator. 6th JFPS Int. Symp. on Fluid Power, pp. 776-770. Tsukuba, Japan, Nov. 7-10 2005.
10. X. Wencan. To measure the mass flow rate characteristic of pneumatic components in series-mounting using sonic velocity exhaust method. Int. Conf. on Fluid Power Transmission and Control, pp.708-711. Hangzhou, China, Mar. 20-22 1989.
11. S. de las Heras. A new experimental algorithm for the evaluation of the true sonic conductance of pneumatic components using the characteristic unloading time. Int. Journal of Fluid Power Vol 2(1):17–24, 2001.
12. S Benchabane, M. Bonis, and J.P. Lecerf. Economic measurement of isopneumatic coefficients. 11th IFK, volume 3, pp. 131–141, Aachen, Germany, 1994.
13. NF49-300:2000. Transmissions pneumatiques - Méthode de caractérisation des coefficients de débit pneumatique par vidange de réservoir, 2000. 22p.
14. K. Kuroshita, Y. Sekiguchi, K. Oshiki, and N. Oneyama. Development of new test method for flow-rate characteristics of pneumatic components. PTMC'04, pp. 243–256, Bath, UK, Sept. 2004.
15. F. Incropera and D. Dewitt. Fundamentals of heat and Mass Transfer. John Wiley and Sons, 2002.
16. F. Det, S. Scavarda, and E. Richard. Simulated and experimental study of charging and discharging of a cylinder using an electropneumatic servovalve. 1st JFPS Int. Symp. on Fluid Power, pp. 199–206. Tokyo, Japan, Mar.13-16 1989.
17. C. Eichelberg. Some new investigations of old combustion engine problems. Engineering, 148(27):463–466, 1939.
18. E.R.G. Eckert and R.M. Drake. Heat and Mass Transfer. McGraw-Hill Book Company, INC., 1959.
19. International Standard ISO 6358. Components using compressible fluids – Determination of flow-rate characteristics, 1989. 15p.