Proceedings of the 7th JFPS International
Symposium on Fluid Power, TOYAMA 2008
September 15-18, 2008

P1-05

OPTIMIZATION OF RELEVANT DESIGN PARAMETERS OF EXTERNAL GEAR PUMPS

Casoli P., Vacca A., Berta G. L.

Department of Industrial Engineering, University of Parma
Viale G. P. Usberti 181/A, 43100 Parma, Italy
(paolo.casoli@unipr.it, andrea.vacca@unipr.it)

ABSTRACT

This paper describes a numerical procedure for the analysis and the optimization of external gear pumps. The Fluid Power group of the University of Parma has implemented a numerical code for the simulation of external gear pumps and motors; the code is named HYGESim and it is based on a lumped parameter approach implemented in AMESim, writing proper submodels in C language. This work is focused on the optimization of the pump design with particular reference to the geometry of the recesses machined on the bushings. The procedure is based on a path search method known as *Steepest Descent* and optimizes the considered parameters starting from a design taken as initial reference. The objective functions defined for the optimization permit to account for the volumetric efficiency, the delivery pressure ripple, the maximum and minimum pressure peaks during the meshing process. An optimal design of the recesses has been proposed and a prototype of a pump, equipped with the proposed geometry of bearing blocks, was tested. Measured data and the comparisons with the experimental results obtained for the stock pump taken as reference are presented in the paper, confirming the potentials of the developed optimization methodology.

KEY WORDS

Gear pump, pressure ripple, design of experiments, response surface methodology.

NOMENCLATURE

A…E	Bushing design parameters
$\hat{y}$	Fitting model
f	Frequency
$\vec{g}$	Gradient function
n	Angular velocity
p	Pressure
Q	Volume flow rate
s	Minimum distance between delivery recess and shaft hole
T	Time
$\vec{u}$	Unit vector
V	Volume
w, z	Weight
θ	Gear angular position
x	Vector of input variable
X	Coded factor
η_v	Volumetric efficiency

Subscripts

av	Average
D	Delivery
Φ	Objective function

i Index
max Maximum
min Minimum

Superscripts
g1 Driver gear
g2 Driven gear

Abbreviations
CV Control volume
OF Objective function
TSV Tooth space volume

INTRODUCTION

The external gear pumps are the most common positive displacement machines in fluid power applications, owing to their simplicity and reliability, despite their unsuitability for variable displacement [1].
Many works, e.g. [2-6], in the open literature are focused on a detailed description of the processes characterizing the machine operation, while others, e.g. [7,8], propose numerical models for the prediction of the gear machine operation. About the application of optimization procedures to hydraulic components, only a few examples appear in the literature as to valves [9], but none has been found for external gear pumps.
The authors have chosen to model the complete machine operation developing a code that provides a detailed evaluation of the flow through the gear machine, allowing, at the same time, the simulation of a complete hydraulic system. The model, named HYGESim (HYdraulic GEar machines Simulator), is based on a discrete parameter approach, and permits the analysis of the flow under a precise characterization of the shape of the teeth profiles, of the recesses and of the axial (gear sides) and radial (between tooth tip and housing) gaps.
This paper reports only a brief description of the HYGESim structure, referring for details to previous works [10-12], being this paper focused on the optimization of the pump design with particular reference to the geometry of the recesses machined on the bushings; as a matter of fact the geometry of the bearing blocks have significant effects on the minimum and maximum pressure peaks during the meshing process, the delivery pressure transients and the volumetric efficiency. The optimization procedure adopted combines Response Surface Methodology and Design Of Experiment (DOE) algorithms, it has been efficiently applied for the optimization of other hydraulic components, as reported in [13,14]. In the present study few objective functions have been considered for the optimization, that permit to account for the main parameters affected by the recesses geometry as previously described.
However this procedure provides reliable results whenever the simulation code evaluates correctly the responses related to all the configurations considered by the optimization process. For this reason the reliability of the HYGESim's results was verified on the basis of experimental data for different prototypes of gear pumps, with particular attention to the parameters that play a relevant role in the optimization problem, such as the flow oscillations at pump delivery (quantified by the measurement of the pressure ripple) and the volumetric efficiency [15]. The target of this work is to propose a new design of the recesses and to test the prototype pump performance comparatively with that of the stock versions of the same pump.

THE SIMULATION CODE (HYGESim)

The structure of the model includes three different parts: 1) the fluid dynamic model; 2) the model for the evaluation of the geometrical features; 3) the module for the evaluation of the actual position of gear's centres. The HYGESim code has been implemented in the AMESim® environment through the development of many submodels written in C language.
The first version of the code was presented in [10], focusing on a description of the fluid dynamic approach; subsequently, improvements of the code are described in [11]. The actual stage of development is reported in [12] where an integrated procedure is presented that permits the user to utilize the CAD 3D drawings directly as an input for the geometrical description of the machine, thus followed by the fluid dynamic simulations in the AMESim® environment. The code permits to consider a variable position of the axes of rotation of both gears during the simulation, as a consequence of the radial forces and torques generated both by the fluid pressure and by the contact between the teeth in the meshing zone. The model also evaluates the eccentricity of the gear shafts respect to the bearings on the basis of either the Ocvirk or the Warner-Sommerfeld models. The fluid dynamic model is the fundamental component of HYGESim, being the other parts developed in accordance with its specifications. It is based on a lumped parameter approach in that the flow through the machine is simulated by subdividing the domain into a number of control volumes (CVs), as displayed in Figure 1; the complete connection framework regarding each CV is quite complex and can be found in [11].

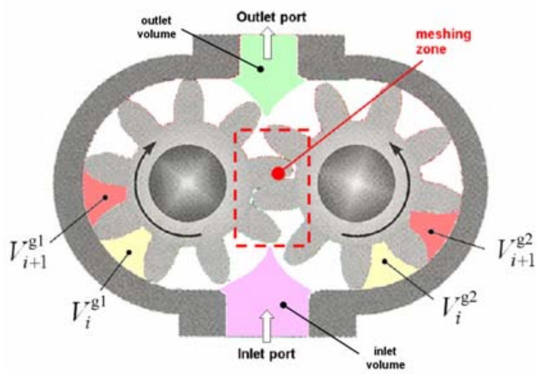

Figure 1 - *Control volumes defined in the fluid dynamic model of the HYGESim*

The actual volume and the connections between adjacent CVs of the framework displayed in Figure 1 are related to the angular position of the gears. For this purpose, a CAD model evaluates all the geometrical

data over a complete revolution of the shaft. Flow rates between different CVs and pressures inside each CV are evaluated according to the Filling & Emptying equations, assuming an isothermal flow. The model accounts for the effects of both gaseous (air release) and vapor cavitation in a simplified manner, by means of a proper relationship between the fluid density and pressure [16]. As concerns the calculation of the leakages (at the tip of each tooth and at gear's lateral surfaces), these are calculated with the modified Poiseuille equation (considering the effect of the shaft speed); in details, the gap between each tooth tip and the casing is determined considering the actual position of the gear centre as a function of the operating conditions.
Thanks to its implementation in the commercial platform AMESim®, HYGESim can be used not only to provide detailed insights into the operation of the machine, but also to simulate the behaviour of the external gear unit in a generic hydraulic system.

OPTIMIZATION PROCEDURE USING DOE AND RSM

The optimization process is based on a numerical procedure implemented in Matlab®, that automatically performs the simulation using the HYGESim code according to the workflow represented in Figure 2.

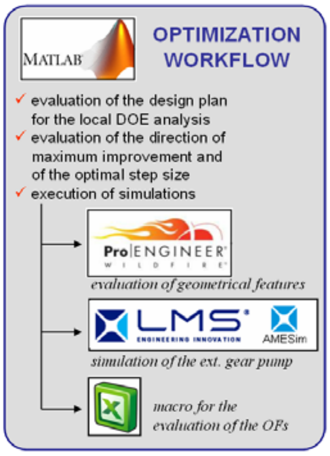

Figure 2 – *The workflow used for the optimization*

The algorithm selected for the optimization is the RSM-*Steepest Descent* method [13]. In particular, the optimization is based on a sequence of line searches in the direction of maximum improvement. The search sequence is continued until there is evidence that the direction chosen does not result in further improvements. The evaluations of the directions of movement and of the proper step sizes are based on the calculations of the value of the defined OF, for each considered geometry.
The procedure approximates the OF using a RMS model, with fitting coefficients that are calculated using DOE technique at each new search sequence. Thus, the shape of the OF is reproduced iteratively with continuous changes in the coefficients of the fitting equations used by the RSM model. This is in contrast to the classic RSM approach conceived for fixed and given functions [17].
The procedure evaluates the coefficients for a fitting model that approximates the OF in a small region around each point considered according to a first order polynomial. A test of adequacy is carried out before moving to a new direction, to determine if the estimated first-order model adequately describes the behaviour of response in the region of factors considered [13]; otherwise a second-order polynomial is considered to approximate the OF near the point considered:

$$\hat{y} = b_0 + x'b + x'Bx \qquad (1)$$

where: x indicates the input variables vector (x' its transpose), b_0 the mean value of responses, b the first-order fitting coefficient vector, and B the 2nd-order fitting coefficient matrix (all terms in matrix B are null for the first order polynomial).
The estimation of the fitting coefficients of Eq. (1) is based on the Ordinary Least Squares method, using the results of a fractional two-level resolution-3 *design* [17].
The gradient of the fitted model is used to determine each new *Steepest Descent* directions. For a single response system, the direction of maximum improvement is determined by the negative of the gradient of $\hat{y}$, Eq. (2) :

$$\vec{g} = \left(\frac{\partial \hat{y}}{\partial X_1}, \frac{\partial \hat{y}}{\partial X_2}, \ldots, \frac{\partial \hat{y}}{\partial X_n} \right) \qquad (2)$$

where X_i represents the dimensionless coded factor related to each input parameter; for example, X_1 is obtained from data of factor A, according to the Eq. (3) :

$$X_1 = \frac{A_i - (A_{\min} + A_{\max})/2}{(A_{\min} - A_{\max})/2} \qquad (3)$$

The coding convention of Eq. (3) has been adopted in order to obtain scale-independent parameter estimates, leading to a more reliable search direction process.
The optimization problem considered in this work is multi-objective: more than one objective function needs to be minimized simultaneously. For this purpose, the procedure follows a weighted priority approach, calculating each search direction by applying assigned weights to the unit vectors provided by the gradients, Eq.(4):

$$\vec{u} = w_{\Phi_1} \cdot \vec{u}_{\Phi_1} + w_{\Phi_1} \cdot \vec{u}_{\Phi_2} + \ldots \qquad (4)$$

where $\vec{u}_{\Phi i} = \dfrac{\vec{g}_{\Phi i}}{\left|\vec{g}_{\Phi i}\right|}$.

Using Eq. (4), the possible differences in the order of magnitude among the typical values of the considered OFs do not affect the effective evaluation of the direction of maximum improvement.

OBJECTIVE FUNCTIONS AND PARAMETERS CONSIDERED IN THE ANALYSIS

The optimization procedure of Figure 2 has been utilized in order to individuate the best geometry of the recesses machined on the bushings. In detail, the input parameters of the optimization problem are represented in Figure 3. The figure represents a typical design of the recesses for a external gear pump and the parameters A, B, C, D, E define a wide range of possible solutions. The region of interest is defined by a proper interval (maximum and minimum values) assigned to each input factor of Figure 3. Obviously, few constraints are required in order to avoid unfeasible configurations. These constraints can be expressed as analytical relationships between the input parameters, and are taken into account by the optimization procedure during the path search process.

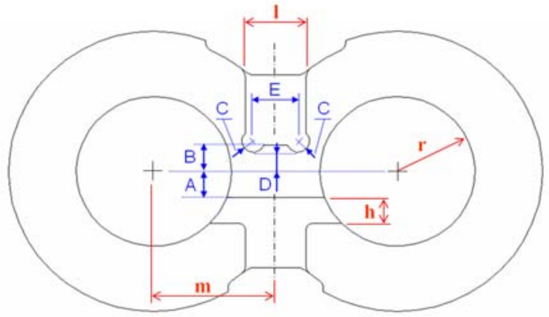

Figure 3 - *Recesses machined on the bushing. Capital letters are used to represent the design parameters of the optimization problem. Other quotes represent the considered constraints*

Specifically, the optimization problem considered in this work requires few geometrical constraints; Table 1 reports the more significant ones.

Table 1 - *Constraints between the factors of Figure 3*

	Constraint
1	$A+D>0$ (This constraint avoid the contact between the recesses)
2	$D \le B$
3	$E+C \ge l$
4	$\sqrt{\left(m-\frac{E}{2}\right)^2+\left(D+\frac{C}{2}\right)^2}-r-\frac{C}{2} \ge s$ (This constraint avoid the contact between the delivery recess and the shaft hole)

As concerns the definition of the objective functions, authors propose a formulations properly to quantify the performance of the considered unit related to the ideal case. As a matter of fact, despite their simple principle of operation, external gear unit are far to be ideal, due to effects mainly related to the displacing action realized by the meshing process. In particular, four different aspects play a crucial role:

1) the amount of transferred flow rate is always lower than the ideal case. This can be quantified by the volumetric efficiency;
2) the reduction of tooth space volume during the first part of the meshing process is responsible of pressure peaks (Figure 4), that can have influence on the flow pulsation at the delivery port. Moreover, they have to be limited to avoid high loading of the gears, mechanical losses and erosion;
3) the geometrical features that characterizes the meshing process (i.e. the course of each TSV, Figure 4) cause oscillations of the flow at pump delivery. This can be quantified by the measurement of the pressure ripple, and it has to be limited as much as possible in order to reduce noise emissions and to enhance the system stability;
4) as pointed out in the detail of Figure 4, the increment of volume that characterizes the second part of the meshing process leads to pressure levels below the suction value, this reduction can cause gas and/or vapour cavitation onset. This phenomenon should be avoided in order to limit mechanical damages and to increase the transferred flow rate.

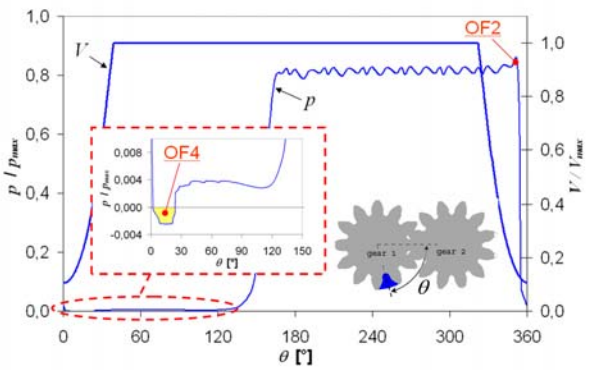

Figure 4 - *Simulated pressure inside TSV; in evidence the first and third objective function*

The optimization problem has been formulated on the basis of the abovementioned effects, in particular an analytical function has been defined to quantify each described aspect. In order to use these functions as OFs for the optimization problem, each function become null in the ideal case. For example, OF_1, related to the volumetric efficiency, is defined as follows:

$$OF_1 = 1 - \eta_v \tag{5}$$

The definition of OF_2 and OF_4 can be described by means of Figure 4: OF_2 is defined considering the peak reached during the meshing process (Figure 4) related to the average delivery pressure level, while OF_4 considers the angular area of the region where the fluid pressure inside the TSV is below the saturation value. In this way instantaneous pressure peaks below the saturation value are not considered directly. This assumption is coherent with the characteristic of the physical phenomenon involved, as a matter of fact both the air release and phase changes are not instantaneous processes.
As concerns the OF_3 its definition is based on the course of the delivery pressure and it takes advantages of the studies [15]. These studies identify the equivalent amplitude of each fundamental term of the FFT course as a parameter suitable for comparisons between different flow ripples. As described in [15] these parameters are calculated on the basis of the Parseval's theorem, summing the amplitude squared of the spectrum. As highlighted by Figure 2, the evaluation of the *OF*s required a proper post processing of data after

each simulation performed using HYGESim. This calculation is carried out using macros written in VisualBasic for Excel®. Moreover, as described in [12], before executing the AMESim® model, HYGESim requires the calculation of the geometrical data using a CAD 2D model (in this case implemented with ProE®). The optimization has been carried out on function OF_{tot}, Eq. (6), in which, the weights z_i have been properly assumed.

$$OF_{tot} = z_1 \cdot OF_1 + z_2 \cdot OF_2 + z_3 \cdot OF_3 + z_4 \cdot OF_4 \qquad (6)$$

RESULTS

The optimization procedure is based on the workflow of Figure 2 using the OF_{tot} defined by Eq. (6). However, in order to define an optimal design of the recesses (Figure 3) for all the typical range of operation of the considered pump, at each step the optimization procedure evaluates the average value of the OF_{tot} obtained for four defined operating conditions, that are displayed in Table 3. The path search algorithm was executed changing the starting point, in this way it has been possible to confirm that the final point correspond to a global minimum in the considered region of interest.

Table 2 – *Operating conditions assumed for the calculation of OF_{tot} at each step of the optimization procedure*

Operating condition	Delivery pressure (bar)	Shaft speed (r/min)
1	100	1000
2	250	1000
3	100	2000
4	250	2000

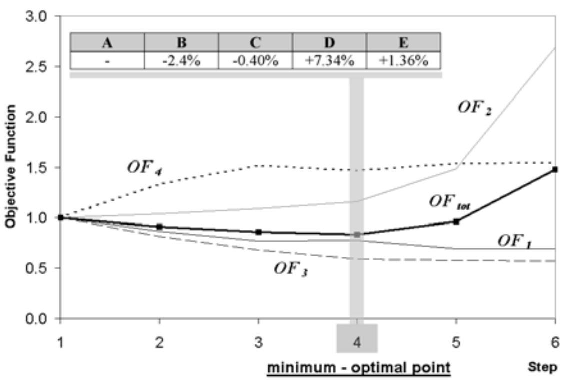

Figure 5 –*Path search sequence for the optimization performed assuming the reference pump as initial point*

Figure 5 reports the details of the path followed by the procedure starting from the configuration of the stock pump taken as reference (Casappa PLP20,11.2 pump). The same figure shows also the dimensionless increment/decrement (as percentage of the reference value) resulting at the end of the procedure, when the final point is reached. From Figure 5 it is possible to notice how the procedure confirms the good design pertinent to the stock pump taken as reference: as a matter of fact starting from its configuration the optimum is reached after only four steps. However, a slight improvement of the performance is achieved considering the new design, as reported by the simulated data provided in Table 3.

Table 3 – *Simulated pump performance: the values are reported as percentage of the reference values (pertinent to the stock pump)*

Operating condition	1	2	3	4
η_v	+1.60	+2.50	+0.70	+0.20
Pressure peaks (g1)	+0.90	+0.23	+5.00	+4.40
Pressure peaks (g2)	+0.14	+0.13	+0.83	+0.76
Pressure ripple	-28.2	-29.9	-13.3	-20.1

Table 3, in accordance with Figure 5, points out how the proposed configuration, respect to the reference pump, improves the volumetric efficiency and reduces the pressure ripple, while a slight (but tolerable) increase of the pressure peak inside the TSVs of both gears is obtained. This also depends on the values of weights z_i assigned for the calculation of OF_{tot}, Eq. (6).
In order to verify the improvement obtained with the new design, a prototype has been realized and tested, and its performance have been compared with the ones pertinent to the stock version of the pump taken as reference. Tests were performed at the Labs. of the Industrial Engineering Dept. of the Univ. of Parma (IED). Figure 6 reports the measured steady-state characteristics, while Figure 7 displays the obtained volumetric efficiencies. The results confirm the numerical predictions: the new pump shows a slight increment of performance respect to reference pump.

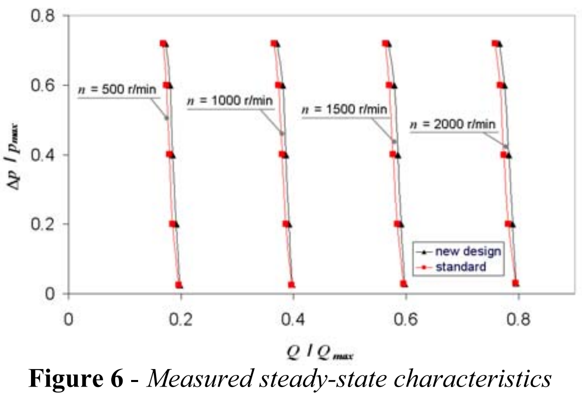

Figure 6 - *Measured steady-state characteristics*

Improvement are more evident observing the course of the delivery pressure ripple, that was measured using the system described in [15]. For all the considered operating conditions a reduction of the oscillations has been observed, as reported in Figure 8 in the time domain. Figure 8 pertain to defined operating condition, but similar improvement were noticed for all the conditions taken into account.

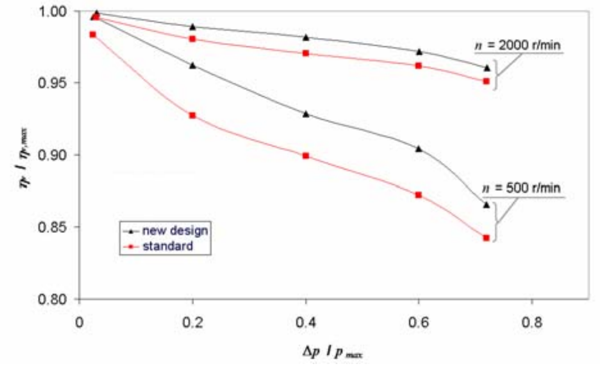

Figure 7 - *Measured volumetric efficiencies*

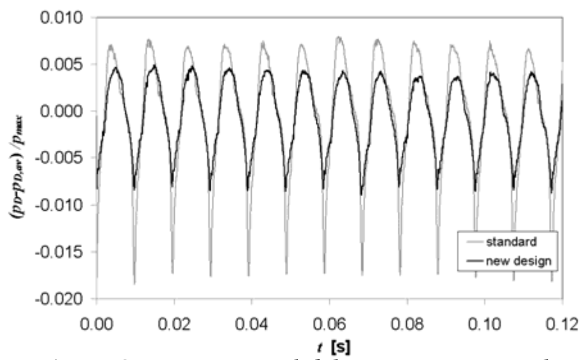

Figure 8 - *Experimental delivery pressure ripple (n=500 r/min $p_{D,av}$ = 100 bar)*

CONCLUSIONS

This work falls in the ambit of the research activity carried out on external gear machines at the IED. In particular this paper presents the numerical optimization of the design of the recesses machined on the bushings. The procedure is based on a path search method known as *Steepest Descent*, it optimizes few defined parameters starting from a design taken as initial reference. The objective functions considered for the optimization permit to account for the volumetric efficiency, the delivery pressure ripple, the cavitation onset and the maximum pressure peak during the meshing process. Considering a stock pump as reference a new design of the recesses has been proposed. Subsequently, a prototype was realized and tested. Measured data, in terms of volumetric efficiency and delivery pressure ripple, confirm the expected improvement of the performance, respect to the stock version of the pump taken as reference.

ACKNOWLEDGEMENTS

The authors would like to thank the R&D division of Casappa S.p.A, for providing founding and supporting this research.

REFERENCES

1. Ivantysyn J., Ivantysynova M., *Hydrostatic Pumps and Motors*, Tech Books Int., 2003, New Delhi, India
2. Mancò S., Nervegna N., *Pressure Transient in an External Gear Hydraulic Pump,* Second JHPS International Symposium on Fluid Power, 2003, Tokyo, Japan.
3. Poy Ferrer M., Codina E., *Suction Capability of Gear Pumps*, 2nd Int. FPNI PhD Symposium, Italy, July 2002.
4. Manring N. D., Kasaragadda S., *The Theoretical Flow Ripple of an External Gear Pump*, ASME Journal of Dynamic Systems, Measurement, and Control., 2003,Vol. 125.
5. Eaton M., Edge K., *Modelling and Simulation of Pressures within the Meshing Teeth of Gear Pumps*, Int. Conf. on Recent Advantages in Aerospace Actuation Systems and Components, June 13-15, 2001, Toulouse, France.
6. Borghi M., Milani M., Zardin B., Paltrinieri F., *The influence of cavitation and aeration on gear pump and motors meshing volume pressures*, Asme-Imece 2006, November 5-10, Chicago, IL (USA).
7. Eaton, M., Keogh, P.S., Edge, K.A., *The modeling, prediction, and experimental evaluation of gear pump meshing pressure with particular reference to aero-engine fuel pumps,* Proc. IMechE Vol.220 Part I: J. System and Control Engineering 2006, pp. 365-379.
8. Wustmann W., Helduser S., Wimmer W. *CFD simulation of the reversing process in external gear pumps,* 6th International Fluid Power Conference, Dresden 31/03-02/04, 2008.
9. Vesely F.., *Design of Pilot Operated Relief Valve by Use of the Simulation*, The 18th Int. Conference on Hydraulics and Pneumatics, 2003, Prague, Czech Republic.
10. Casoli P., Vacca A., Franzoni G., *A numerical model for the simulation of external gear pumps,* The sixth JFPS International Symposium on Fluid Power, Tsukuba, Japan, Novembre 07-11 2005.
11. Casoli P., Vacca A., Berta G.L., *A Numerical Model for the Simulation of Flow in Hydraulic External Gear Machines*, Power Transmission and Motion Control, 13-15 September 2006, University of Bath (GB).
12. Casoli, P., Vacca, A., Berta, G.L., *Potentials of a Numerical Tool for the Simulation of Flow in External Gear Machines,* The Tenth Scandinavian International Conference on Fluid Power, May 21-23, 2007, Tampere, Finland
13. Vacca, A., Cerutti, M., *Analysis and Optimization of a Two-Way Valve Using Response Surface Methodology,* Int. Journal of Fluid Power, Vol. 8, n. 3, November 2007, pp. 43-59.
14. Vacca A., Greco M. Fornaciari A., *A Methodology for the Development of a Relief, Anti-Shock and Anti-Cavitation Cartridge Valve,* 6th International Fluid Power Conference, Dresden 31/03-02/04 2008.
15. Casoli P., Vacca A., Franzoni G., Guidetti M., *Effects of Some Relevant Design Parameters on External Gear Pumps Operating: Numerical Predictions and Experimental Investigations,* 6th International Fluid Power Conference (pp. 469-483 vol. 2), Dresden, 31/03-02/04 2008.
16. Casoli P., Vacca A. Franzoni G., Berta G.L., *Modelling of fluid properties in hydraulic positive displacement machines,* Simulation Modelling Practice and Theory 14, 2006, pp. 1059–1072, Elsevier.
17. Montgomery D.C., *Design and Analysis of Experiments.* 1997, J. Wiley & Sons