

ESTIMATION OF A CRITICAL PRESSURE RATIO OF A PNEUMATIC PIPE-LINE AND ITS SIMULATION

Katsushi SATOH*, Yukio KAWAKAMI* and Kazuo NAKANO**

*Department of Machinery and Control Systems, College of Systems Engineering
Shibaura Institute of Technology
307 Fukakusa, Minuma-ku, Saitama, 337-8570 JAPAN
(E-mail: i023441@sic.shibaura-it.ac.jp, kawakami@shibaura-it.ac.jp)

**Tokyo Institute of Technology
1-26-28 Izumihon-cho, Komae-shi, Tokyo, 201-0003 JAPAN
(E-mail: nakanokazu@jcom.home.ne.jp)

ABSTRACT

We considered the so-called Fanno flow as a model of flow through pneumatic pipe lines with a constant diameter. The flow is one dimensional, steady and adiabatic. The friction coefficient is constant. We succeeded to express the critical pressure ratio of the flow which is choking at the exit of the pipe line by three parameters, the pipe diameter, the pipe length and the friction coefficient. We compared the theoretically estimated critical pressure ratio to the experimental results. The results showed the good agreements except the case of the short pipe lines less than 10 cm. It was confirmed that our method of determination of the ratio was very useful for the flow calculation using ISO choke equation. With these studies we established the simulation program for the Fanno flow on MATLAB_Simulink and show that the program was easily applied to connected pipelines in series.

KEY WORDS

Pneumatic, Pipeline, Simulation, Friction, Choking

NOMENCLATURE

b : Critical pressure ratio
 c : Speed of sound
 D : Inner diameter
 f : Friction factor
 K : Loss coefficient
 L : Length of pipe
 Q : Volumetric flow rate
 q_m : Mass flow rate
 R : Gas constant of air
 T : Temperature
 M : Mach number
 p : Static pressure
 v : Axial speed of flow
 κ : Specific heat ratio
 ρ : Density of air

(Subscripts)

1 : Up stream state point
2 : Down stream state point
a : Approximate value
c : Critical condition
e : Exit

INTRODUCTION

It is the one of important problems in the designing pneumatic systems to estimate the pressure drop in pipe lines with losses. Ignoring the compressibility of the air, the estimation becomes simple but the accuracy is not enough for the flow of the large Mach number. The conventional analysis of the Fanno flow^{[1],[2],[3],[4]} is applicable for the air flow through the uniform adiabatic rigid pipe with losses. But it has the short-coming that the analytical solution to represent the pressure drop as a

function of the mass flow rate is not available and so the numerical method is inevitable in the analysis.

In this paper we introduce the analytical forms of the approximate solutions for Fanno flow and present the simulation programs on MATLAB Simulink^[5] for the steady, adiabatic, one dimensional pneumatic flow in uniform pipes with losses.

BASIC EQUATIONS

Generally, the basic equations for a pneumatic pipe line are assigned as follows.

$$\frac{d\rho}{\rho} + \frac{dv}{v} = 0 \quad (1)$$

$$v dv = -\frac{dp}{\rho} - \frac{4f}{D} dx \frac{v^2}{2} = -\frac{dp}{\rho} - dK \frac{v^2}{2} \quad (2)$$

$$v dv + c_p dT = 0 \quad (3)$$

$$\frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T} \quad (4)$$

Four unknown p , T , v and ρ are determined from these equations for given D , L , f and the boundary conditions.

APPROXIMATIONS FOR THE FANNO FLOW

The friction losses of the flow lead to the pressure drop and the velocity increase along the flow. When the Mach number M_e at the exit of the pipe approaches to the unity, the choking of the flow occurs. The state variables and parameters on the pipe-line are specified as shown in Figure 1. Integrating the basic equations as shown in the references, the loss coefficient K_{12} and the pressure ratio between the state point 1 and 2 is expressed as the function of the Mach numbers as followings.

$$K_{12} = \frac{4fL}{D} \left[1 - \left(\frac{M_1}{M_2} \right)^2 + \frac{\kappa + 1}{2\kappa} \ln \left\{ \frac{M_1^2}{M_2^2} \cdot \frac{2 + (\kappa - 1)M_2^2}{2 + (\kappa - 1)M_1^2} \right\} \right] \quad (5)$$

$$\frac{p_2}{p_1} = \frac{M_1}{M_2} \sqrt{\frac{2 + (\kappa - 1)M_1^2}{2 + (\kappa - 1)M_2^2}} \quad (6)$$

Define the approximate Mach number ratio $(M_1/M_2)_a$ as shown in the following equation.

$$\frac{\kappa + 1}{2 + (\kappa - 1)(M_1/M_2)_a^2} = \frac{2 + (\kappa - 1)M_2^2}{2 + (\kappa - 1)M_1^2} \quad (7)$$

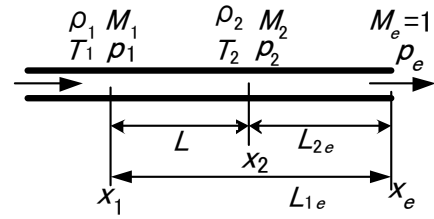


Figure 1 Pipe line and states

This approximation simplifies Eq. (6) as follows,

$$\frac{p_2}{p_1} = \left(\frac{M_1}{M_2} \right)_a \sqrt{\frac{2 + (\kappa - 1)(M_1/M_2)_a^2}{\kappa + 1}} \quad (8)$$

Solving Eq. (8), we get the approximate Mach number ratio as the function of the pressure ratio.

$$\left(\frac{M_1}{M_2} \right)_a = \frac{1}{\sqrt{\kappa - 1}} \left\{ \sqrt{1 + (\kappa^2 - 1) \left(\frac{p_2}{p_1} \right)^2} - 1 \right\}^{1/2} \quad (9)$$

Substituting the approximate Mach number ratio $(M_1/M_2)_a$ and Eq. (7) into Eq. (5), the loss coefficient K_{12} is expressed as follows.

$$K_{12} = \frac{1 - (M_1/M_2)_a^2}{\kappa M_{1a}^2} + \frac{\kappa + 1}{2\kappa} \ln \left(\frac{M_1}{M_2} \right)_a^2 + \frac{\kappa + 1}{2\kappa} \ln \left\{ \frac{\kappa + 1}{2 + (\kappa - 1)(M_1/M_2)_a^2} \right\} \quad (10)$$

M_{1a} is defined as the approximate value of M_1 which fulfils Eq. (10) and expressed by the following.

$$M_{1a} = \frac{\sqrt{\left\{ 1 - (M_1/M_2)_a^2 \right\} / \kappa}}{\sqrt{\frac{4fL}{D} - \frac{\kappa + 1}{\kappa} \ln \left(\frac{p_1}{p_2} \right) - \frac{2(\kappa + 1)}{\kappa} \ln \left(\frac{M_1}{M_2} \right)_a}} \quad (11)$$

From Eq. (9) and Eq. (11) we can get the analytical equation which expresses the approximate Mach number M_{1a} as the function of the pressure ratio p_2/p_1 . Solving Eq. (6) and Eq. (11), the approximate Mach number M_{2a} at the state point 2 is obtained.

$$M_{2a} = \left[\frac{\sqrt{1 + (\kappa - 1)M_{1a}^2} \left\{ 2 + (\kappa - 1)M_{1a}^2 \right\} (p_1/p_2)^2 - 1}{\kappa - 1} \right]^{1/2} \quad (12)$$

The sound speed at the state point 1 is given by the next.

$$c_1 = \sqrt{\kappa R T_1} \quad (13)$$

The mass flow rate is given by the next.

$$q_m = \frac{\pi D^2}{4} \rho_1 v_1 = \frac{\pi D^2}{4} \cdot \sqrt{\frac{\kappa}{R}} \cdot \frac{p_1}{\sqrt{T_1}} M_{1a} \quad (14)$$

It was presented that the approximate mass flow rate of Fanno flow could be expressed in the analytical form from the given pressure at arbitral two cross sections.

CHOKED FLOW

When the flow is choked at the exit of the pipe, the loss coefficient K between the state point 1 and the exit is obtained from Eq. (1), substituting $M_e=1$ to M_2 .

$$K = \frac{1 - M_{1c}^2}{\kappa M_{1c}^2} + \frac{\kappa + 1}{2\kappa} \ln \left\{ \frac{(\kappa + 1)M_{1c}^2}{2 + (\kappa - 1)M_{1c}^2} \right\} \quad (15)$$

,where M_{1c} is the Mach number at the state point 1 for the flow choked at the exit. The value of the loss coefficient K is shown in Figure 2. The computation of the loss coefficient K for the given the Mach number M_{1c} is straight forward but the inverse has to use a trial and error method. We proposed the functions to compute the approximate Mach number M_{1ca} from the loss coefficient as follows.

In the range of $0 \leq K \leq 1.3$

$$M_{1ca} \cong 0.95 \times \exp(-0.728K^{0.415}) + 0.05 \quad (16)$$

The M_{1ca} is compared with M_{1c} as shown in Figure 3.

In the range of $0.5 \leq K \leq 25$

$$M_{1ca} \cong 0.9 \times \exp(-0.787K^{0.386}) + 0.1 \quad (17)$$

Eq. (16) and Eq. (17) coincides at the value $K=0.9335$.

In the range of $25 \leq K \leq 120$

$$M_{1ca} \cong 0.4 \times \exp(-0.29K^{0.475}) + 0.052 \quad (18)$$

The relative errors $(M_{1ca} - M_{1c}) / M_{1c}$ for Eq. (16), Eq. (17) and Eq. (18) are less than 1% in the respective ranges. The pressure ratio p_2/p_1 for the choked flow is defined as the critical pressure ratio b . Substituting M_{1ca} into Eq. (8), the critical pressure ratio b is obtained as shown followings.

$$b = \frac{p_{2c}}{p_{1c}} = M_{1ca} \cdot \sqrt{\frac{2 + (\kappa - 1)M_{1ca}^2}{\kappa + 1}} \quad (19)$$

From Eq.(15) and Eq. (19), we can plot the b as a function of K as shown in Figure 6.

The critical pressure ratio b is highly affected by the value of the loss coefficient K in the lower range than 20.

The loss coefficient $K=0$ means the non-viscous case and the corresponding critical pressure ratio b is the unity. It shows the Mach number M_1 at the state point 1 is equal to 1.

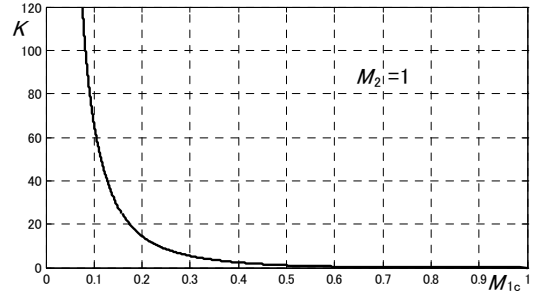


Figure 2 Loss coefficient K

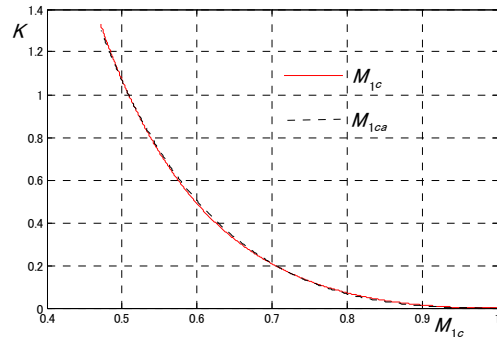


Figure 3 M_{1ca} and M_{1c} ($0 \leq K \leq 1.3$)

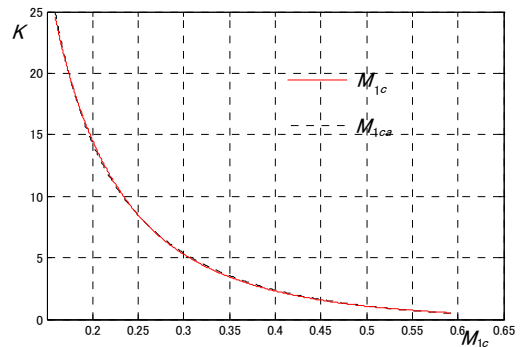


Figure 4 M_{1ca} and M_{1c} ($0.5 \leq K \leq 25$)

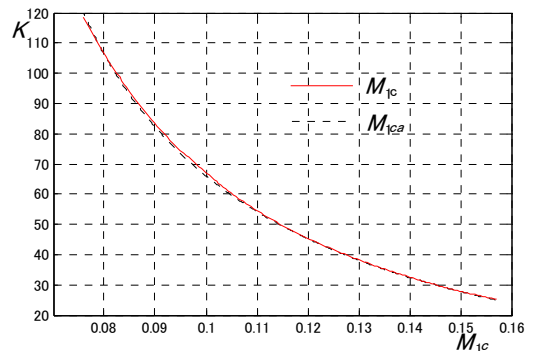


Figure 5 M_{1ca} and M_{1c} ($25 \leq K \leq 120$)

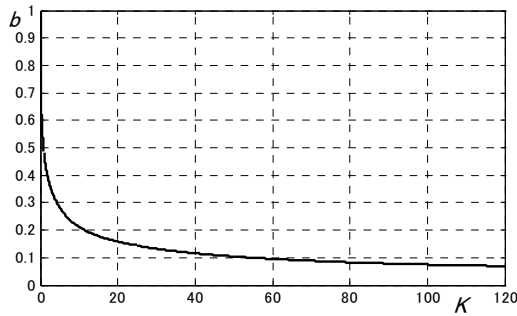


Figure 6 Critical pressure ratio b

SIMULATION PROGRAM ON MATLAB

Now, it became practicable to solve the unknown variables in the Fanno flow by the direct computation of the above mentioned equations under the given conditions. We adopted MATLAB as the one of the simulation tools to simulate the one dimensional steady flow with losses through adiabatic uniform pipes of finite lengths.

As the first step of the simulation, the parameters and the boundary conditions shown in Figure 7 have to be given. The next steps are creating blocks and subsystems to compute equations by making use of the Simulink as shown in Figure 8. The left three ports in Figure 8 are the given states at boundaries and the right five ports give outputs. The details of five subsystems with labels in Figure 8 are shown in Appendix. The subsystem p2/p1 determines p_{2c} at the choking condition by making use of Eq. (16), (17) and (18), and gives the pressure ratio p_2/p_1 . The subsystem Mach1 computes Eq. (9), (10) and (11) to get M_{1a} from the obtained the pressure ratio p_2/p_1 . The subsystem Mach2 outputs M_{2a} from M_{1a} and p_2/p_1 by computing Eq. (12). The subsystem q2 gives the mass flow rate q_{m1} by computing Eq. (14). The subsystem T2 computes T_2 from T_1 , M_{1a} and M_{2a} by making use of Eq. (20) which is deduced from the fundamental equations for the Fanno flow [1].

$$T_2 = \frac{2 + (\kappa - 1)M_1^2}{2 + (\kappa - 1)M_2^2} T_1 \quad (20)$$

EXPERIMENTS AND SIMULATIONS

The flow rate of the choked flow is given by Eq. (21).

$$q_{mc} = \frac{\pi D^2}{4} \cdot \sqrt{\frac{\kappa}{R}} \cdot \frac{p_1}{\sqrt{T_1}} M_{1ca} \quad (21)$$

The nondimensional flow rate q_n is defined by Eq. (22).

$$q_n = q_m / q_{mc} \quad (22)$$

Figure 9 shows the theoretical curves and the marks

of the experimental results [6] which were done at SMC Co. The factor f was chosen to fit the experimental results. The relation between the critical pressure ratio b and the ratio L/D is obtained by our simulation method and it is shown as the one solid line in Figure 10.

Circle marks and asterisk marks are the estimated values of the critical pressure ratio which are based on the results of the numerical flow analysis [7] and the experimental results [6] done at SMC Co. respectively.

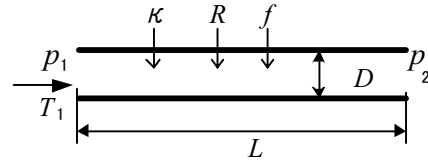


Figure 7 Parameters and boundary conditions of pipe

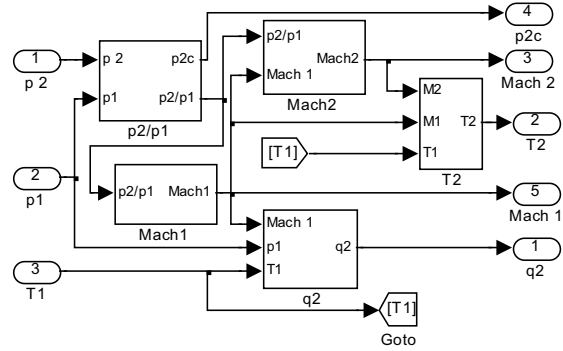
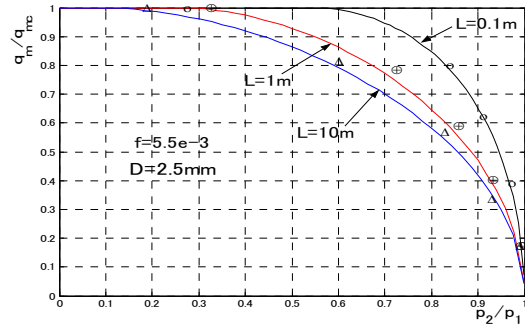
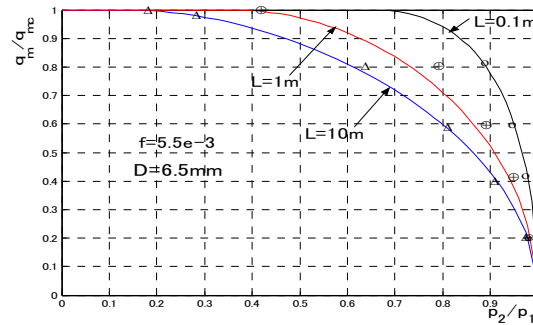


Figure 8 Simulink block diagram of pneumatic lines



(a) $D = 2.5 \text{ mm}$



(b) $D = 6.5 \text{ mm}$

Figure 9 Nondimensional flow rate

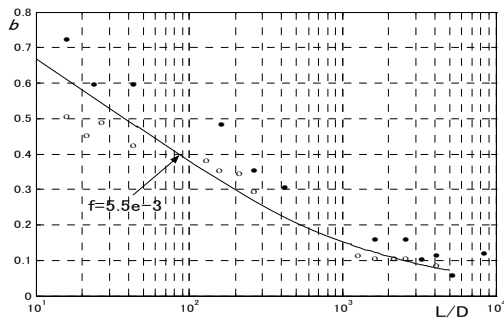


Figure 10 Critical pressure ratio b

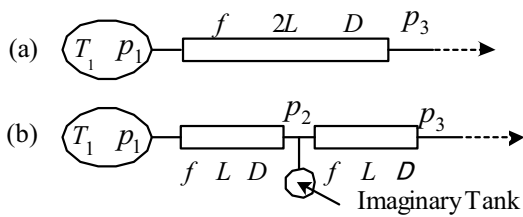


Figure 11 Series connection of pipe lines

The simulation of flows through the series connection of pipes as shown in Figure 11 can be realized by the straight-forward connection of Simulink blocks of each line. In this case we put imaginary tanks at each junction to avoid causal conflicts in computations.

The parameters shown in Figure 11 are $f=0.0055$, $D=4.01\text{mm}$, $L=4.83\text{m}$. The nondimensional flow rate q_n computed as the single line (a) is shown as the solid curve in Figure 12. The same flow rate q_n computed as the connection of two same lines (b) is shown as the diamond marks. The difference between both simulation results is negligible for practical uses. The asterisks and circles shown in Figure 12 are experimental results done at Shibaura Institute of Technology for the single line (a).

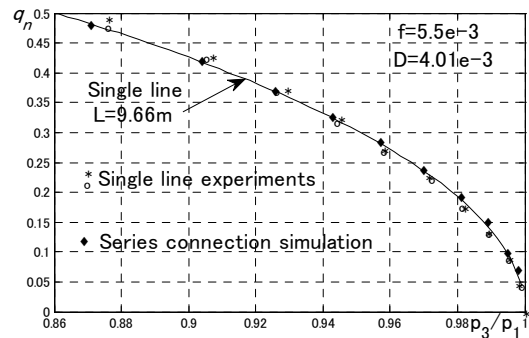


Figure 12 Simulation and experiments

CONCLUSION

It was confirmed that our simulation method was excellent enough to estimate the flow characteristics of pipe lines for designing pneumatic systems. Especially, it is practically useful that the critical pressure ratio b can be expressed by the loss coefficient of the pipe line and the simulation of flow for connected lines in series is straight-forward.

ACKNOWLEDGEMENT

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APPENDIX

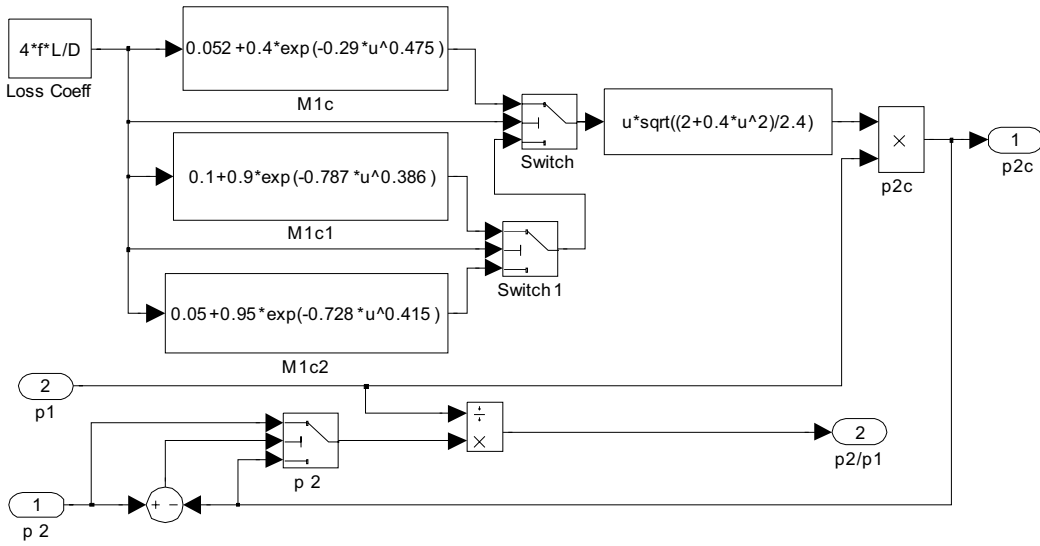


Figure A1 Subsystem p2/p1

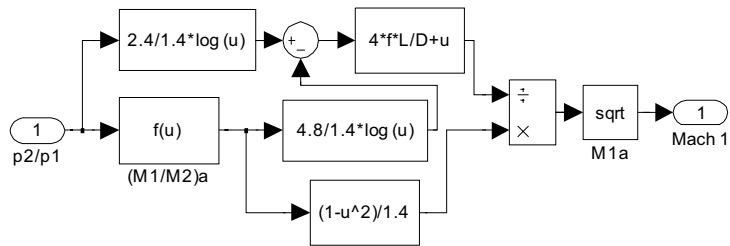


Figure A2 Subsystem Mach1

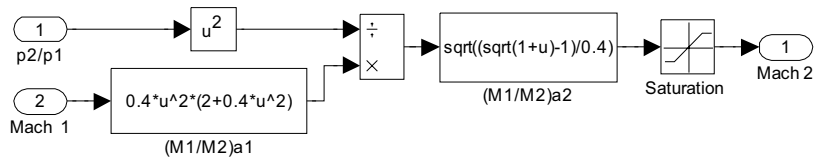


Figure A3 Subsystem Mach2

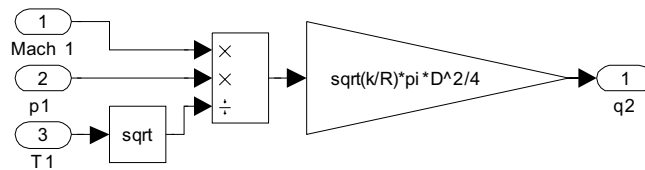


Figure A4 Subsystem q2

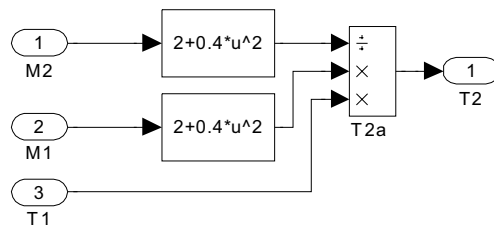


Figure A5 Subsystem T2