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# NONLINEAR MODELING AND CONTROL DESIGN OF ELECTRO-HYDROSTATIC ACTUATOR

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## ABSTRACT

In this paper, a typical architecture of Electro-Hydrostatic Actuator (EHA) is described and modeled by the block diagram based on mathematic equations, which is a nonlinear accuracy model. The single PID, cascade PID and state feedback controllers are respectively designed and applied to this model for comparison analysis that focused on system stability, stiffness and dynamic characteristics. It is proven that the state feedback controller along with dynamic pressure feedback strategy could efficiently improve both static and dynamic performance.

## KEY WORDS

EHA, Block diagram model, Single PID, Cascade PID, State feedback

## NOMENCLATURE

$A$ : Piston active area [m$^2$]
$B$ : Oil bulk modulus [N/ m$^2$]
$D$ : Pump displacement [m$^3$/rad]
$F_c$ : Coulomb friction [N]
$F_s$ : Maximum static friction of piston [N]
$F_{ex}$ : External load force [N]
$J$ : Total inertia of motor and pump [Kg·m$^2$]
$K_c$ : Motor speed constant [v/(rad/s)]
$K_t$ : Motor torque constant [N·m/A]
$K_{elp}$ : Pump external leakage coefficient [(m$^3$/s)/Pa]
$K_{ilp}$ : Pump internal leakage coefficient [(m$^3$/s)/Pa]
$K_{ilj}$ : Hydraulic jack internal leakage coefficient [(m$^3$/s)/Pa]
$K_{vism}$ : Motor viscous coefficient [N·m/(rad/s)]
$K_{visp}$ : Piston viscous coefficient [N/(m/s)]
$k$ : Polytropic exponent [-]
$L$ : Motor inductance [H]
$M$ : Total mass of piston and load [Kg]
$R$ : Motor resistance [Ohm]
$V_{10}$ : Initial volume in chamber 1 [m$^3$]
$V_{20}$ : Initial volume in chamber 2 [m$^3$]
$X_t$ : Output displacement of actuator [m]
$\omega$ : Rotational speed of motor [rad/s]

## INTRODUCTION

The demand for conventional hydraulic actuation is gradually decreasing due to its limitations such as: low energy efficiency, leakage, noise, low maintainability. The Power-By-Wire (PBW) technology is becoming an attractive direction of future airborne actuation system. A PBW flight control system would simplify the secondary power generation, eliminate the need for a central hydraulic power supply, and replace the hydraulic pipes by electric power cables. As a result, the reliability,


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survivability, efficiency and maintainability of the aircraft would be greatly improved.

The Electro-Hydrostatic Actuator (EHA) is one kind of PBW actuator that uses the hydraulic pump to transfer the rotational motion of electrical motor to the actuator output. EHA is based on the principle of closed-circuit hydrostatic transmission, so that, there are no requirements for oil reservoir or electro-hydraulic servo-valves.

A lot of research papers have modeled the EHA system by transfer functions in the past [1,2,3], however, it is a linear modeling method that has some disadvantages in description of realistic EHA: neglecting the refeeding circuit which contains some nonlinearity; simplifying the friction, especially the static friction; supposing that all the initial conditions are zero. To solve these problems, the block diagram model of EHA is established; furthermore, some different control methods are designed and compared in this paper.

## EHA ARCHITECTURE DESIGN

There are several architectures of EHA: EHA with fixed pump displacement and variable motor speed (FPVM), EHA with variable pump displacement and fixed motor speed (VPFM), EHA with variable pump displacement and variable motor speed(VPVM). Nowadays, the FPVM-EHA (Fig.1) is more popular for its simple structure and efficiency. In this system, a bi-directional pump rotates in variable speed and directions given by electric motor. As a result, the oil flow and supply pressure are variable to drive the symmetrical actuator.

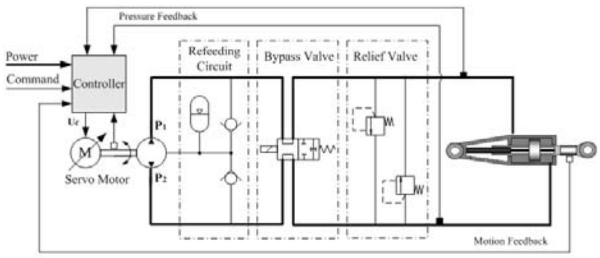


Figure 1 Architecture of FPVM-EHA

## EHA MODELING

### Modeling of DC Motor

The Brushless DC Motor (BLDCM) is chosen as the EHA driving motor. This is mostly due to its high reliability which is very important for airborne actuation. The mathematic equations of BLDCM are:

$$\begin{cases} U_c = E + L\dfrac{di}{dt} + Ri \\ E = K_c\omega \\ T_e = K_t i \\ T_e = J\dot{\omega} + T_f + T_l \end{cases} \tag{1}$$

According to Eq.(1), a block diagram model of motor is gotten as Fig.2 by Simulink. The part in dashed is current protection that could be realized by software.

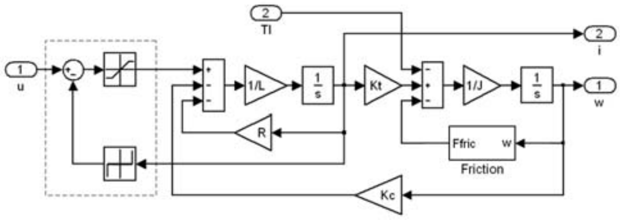


Figure 2 Block diagram model of motor

### Modeling of Pump

The flow equation of pump outlet is:

$$Q_1 = D\cdot\omega - K_{ilp}(P_1 - P_2) - K_{elp}(P_1 - P_{ac}) \tag{2}$$

The flow equation of pump entry is:

$$Q_2 = D\cdot\omega - K_{ilp}(P_1 - P_2) + K_{elp}(P_2 - P_{ac}) \tag{3}$$

Where $P_{ac}$ is the pressure of accumulator. The diagram model of pump is shown in Fig. 3.

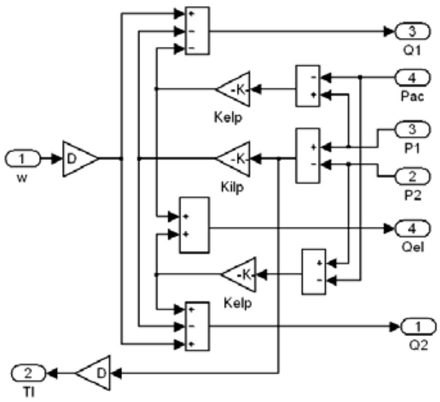


Figure 3 Block diagram model of pump

## Modeling of Refeeding Circuit

To keep the closed-circuit of EHA, a refeeding circuit composed of accumulator and check valves is necessary. The schematic diagram is shown in Fig. 4.

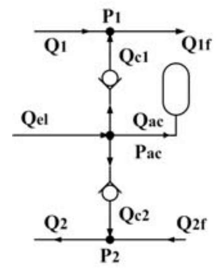


Figure 4 Schematic diagram of refeeding circuit

The flow equations of refeeding circuit are:

$$\begin{cases} Q_{ac} = Q_{el} - Q_{c1} - Q_{c2} \\ Q_{1f} = Q_1 + Q_{c1} \\ Q_{2f} = Q_2 - Q_{c2} \end{cases} \tag{4}$$

Where $Q_{el}$ is the external leakage of pump, $Q_{c1}$, $Q_{c2}$ are the flow of check valves depending on $(P_1 - P_{ac})$ and $(P_2 - P_{ac})$. The relationship between $Q_{ac}$ and $P_{ac}$ could be described as follows:

$$P_{ac} = P_{aci} V_{gasi}^{k} / (V_{gasi} - \int Q_{ac} dt)^{k} \tag{5}$$

Where $P_{aci}$ is the initial pressure of accumulator, $V_{gasi}$ is the initial volume of gas, $k$ is the polytropic exponent of gas within the range from 1.0 to 1.4.

The block diagram model of refeeding circuit and accumulator are shown in Fig. 5 and Fig. 6.

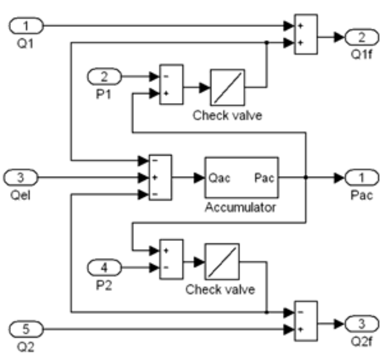


Figure 5 Block diagram model of refeeding circuit

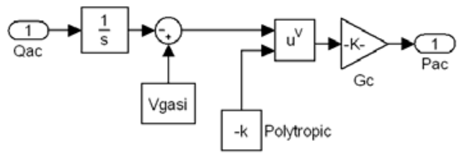


Figure 6 Block diagram model of accumulator

## Modeling of Hydraulic actuator

EHA requires a symmetrical actuator in order to ensure flow balance between the actuator and the pump. The hydraulic jack is divided into two working chambers by the piston [4].

The flow of oil-in chamber could be described by the following equation:

$$Q_{1f} = A\dot{x}_t + \frac{V_{10} + Ax_t}{B}\dot{P}_1 + K_{ilj}(P_1 - P_2) \tag{6}$$

The flow of oil-out chamber could be described by the following equation:

$$Q_{2f} = A\dot{x}_t - \frac{V_{20} - Ax_t}{B}\dot{P}_2 + K_{ilj}(P_1 - P_2) \tag{7}$$

In such a symmetrical actuator, the initial volume $V_{10}$ and $V_{20}$ are of the same value.

The load force balance equation of the piston is:

$$A(P_1 - P_2) = M\ddot{x}_t + F_{ex} + F_{fric} \tag{8}$$

Where $F_{fric}$ is the friction which would be described later. The diagram model of hydraulic jack and piston are respectively shown in Fig. 7 and Fig. 8:

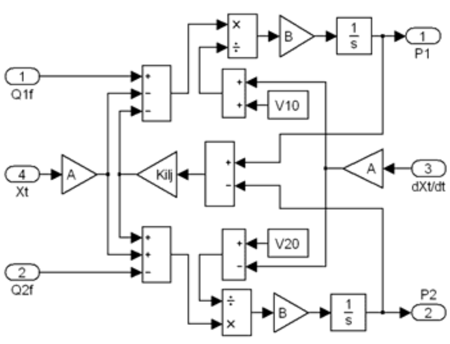


Figure 7 Block diagram model of hydraulic jack



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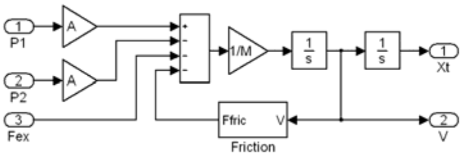


Figure 8 Block diagram model of piston

**Modeling of Friction**
The friction model is given as follows [5]:

$$F_f(\dot{x}_t) = [F_c + (F_s - F_c)\cdot e^{-|\dot{x}_t|/\alpha} + K_{vis}\,|\dot{x}_t|]\cdot sign(\dot{x}_t) \quad (9)$$

Where $F_c$ is the Coulomb friction, $(F_s - F_c)\cdot e^{-|\dot{x}_t|/\alpha}$ is the Stribeck friction, and $K_{vis}\,|\dot{x}_t|$ is the viscous friction. For improvement, $sign(\dot{x}_t)$ could be replaced by $\tanh(\dot{x}_t/\beta)$ to make the model continuous:

$$F_f(\dot{x}_t) = [F_c + (F_s - F_c)\cdot e^{-|\dot{x}_t|/\alpha} + K_{vis}\,|\dot{x}_t|]\cdot \tanh(\dot{x}_t/\beta) \quad (10)$$

Where $\alpha$, $\beta$ are the reference speeds approximately within the range from 0.001m/s to 0.01m/s. The diagram model of the friction is shown in Fig. 9.

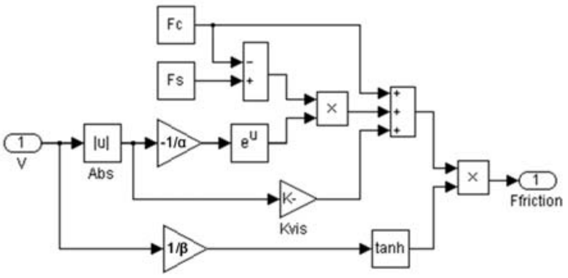


Figure 9 Block diagram of friction

**Overall EHA System**
Based on the above sub-models, an open-loop FPVM-EHA could be gotten as Fig.10. All the sub-models are enveloped and connected with each other by the defined input and output ports. The EHA parameters are given in Table 1.

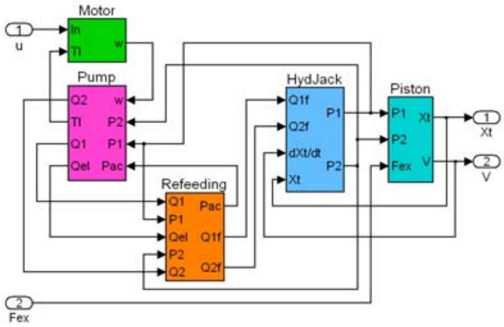


Figure 10 EHA open-loop system

Table 1 EHA Parameters

| Symbol | value | Symbol | value |
|---|---|---|---|
| $L$ [H] | 2.3e-3 | $F_s$ [N] | 25 |
| $R$ [Ohm] | 1.5 | $F_c$ [N] | 15 |
| $K_t$ [N·m/A] | 0.2 | $T_s$ [N·m] | 0.5 |
| $K_c$ [v/(rad/s)] | 0.2 | $T_c$ [N·m] | 0.3 |
| $J$ [Kg·m$^2$] | 1.2e-3 | $V_{10}, V_{20}$ [ml] | 152 |
| $K_{vism}$ [N·m/(rad/s)] | 4.2e-4 | $B$ [N/m$^2$] | 6.5e8 |
| $K_{ilp}$ [(m$^3$/s)/Pa] | 1e-13 | $A$ [cm$^2$] | 19 |
| $K_{elp}$ [(m$^3$/s)/Pa] | 1e-13 | $M$ [Kg] | 2000 |
| $D$ [ml/r] | 1.2 | $V_{gasi}$ [ml] | 150 |
| $K_{ilj}$ [(m$^3$/s)/Pa] | 1e-13 | $P_{aci}$ [MPa] | 2.5 |
| $K_{visp}$ [N/(m/s)] | 150 | $V_{oili}$ [ml] | 150 |
| $k$ | 1.3 | | |

## CONTROL DESIGN AND COMPARISONS

The EHA is a unique device with some complex characteristics due to the combination of electrical and hydraulic components. To obtain desire performance, the single PID, cascade PID, and state feedback control methods are analyzed and estimated.

**Single PID Control**
Single PID control is popular for its simple structure. Fig.11 gives the position step response, where the input is 20mm and an external force of 20000N is acting on actuator at 2.5s.

The static error caused by the external load indicates the low stiffness of this close-loop system. Moreover, there are some oscillations in pressure response due to the low hydraulic damping. For improvement, the cascade PID control may be a better choice.

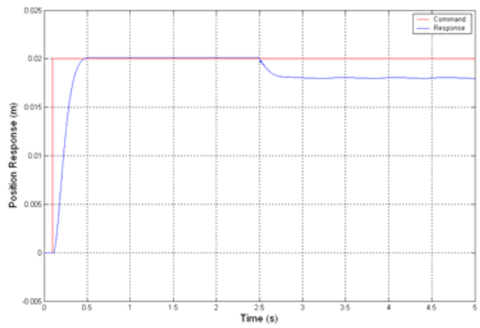


Figure 11 Step response of single PID controlled EHA

**Cascade PID Control**

The cascade control could detect and compensate the disturbances in the inner loop before they affect the outer variable and speed up the system because the inner loop could use a faster controller [6].

To compose the cascade controller, from inside to outside, there are current loop of motor, speed loop of motor and position loop of EHA. Considering the strong integration in the current loop, the anti-integration saturation method is necessary.

Fig.12 gives the position response. The rise time is reduced to 0.25s, and the static error due to external load is negligible. Compared to single PID control, the cascade control efficiently improves the system's rapidity and stiffness.

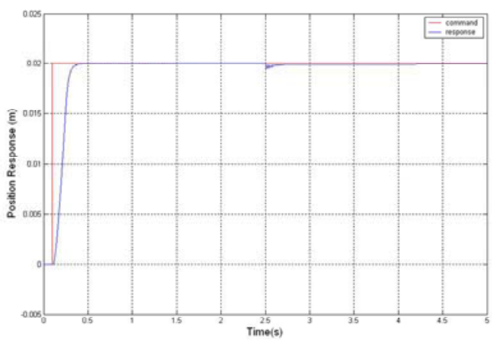


Figure 12 Step response of cascade controlled EHA

However, in view of Fig.13, the pressure oscillations seem to be critical. The frequency is about 36Hz which is close to EHA's natural frequency:

$$f_N = \sqrt{\frac{2 \cdot B \cdot A^2}{M \cdot V_{10}}} = 231.6 rad/s = 36.8 Hz \qquad (11)$$

The usual solution is using the velocity or pressure feedback which means another new loop to the system and becoming difficult to design the parameters in different loops to achieve global high performance.

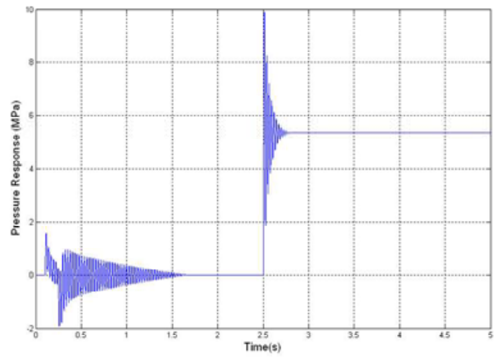


Figure 13 Pressure response of cascade controlled EHA

**State Feedback Control**

Consider the EHA as a fifth-order system with the following state variables:

$$\boldsymbol{X} = \begin{bmatrix} i & \omega & P_f & x_t & \dot{x}_t \end{bmatrix}^T \qquad (12)$$

All these variables are measurable, which is important for the realization of a full state feedback controller. According to Eq.(1), (2), (3), (6), (7), (8), the state equation of EHA could be described in matrix form:

$$\dot{\boldsymbol{X}} = \begin{bmatrix} -\frac{R}{L} & -\frac{K_c}{L} & 0 & 0 & 0 \\ \frac{K_t}{J} & -\frac{K_{vism}}{J} & -\frac{D}{J} & 0 & 0 \\ 0 & \frac{2BD}{V_{10}} & -\frac{2BK_l}{V_{10}} & 0 & -\frac{2AB}{V_{10}} \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{A}{M} & 0 & -\frac{K_{visp}}{M} \end{bmatrix} \boldsymbol{X} + \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -\frac{1}{M} \end{bmatrix} \boldsymbol{U} \qquad (13)$$

Input vector: $\boldsymbol{U} = \begin{bmatrix} u & F_{ex} \end{bmatrix}^T$

Output vector: $\boldsymbol{Y} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix} \boldsymbol{X}$

Fig.14, 15 are the position and pressure responses of this system. Compared to cascade PID control, it is a more rapid system and the oscillation is obviously reduced. However, there is an unacceptable static error when loading. The system has to maintain a certain static error to produce enough input to counteract the input loss due to pressure feedback.

To solve this problem, a high-pass filter could be applied


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to the pressure feedback loop to form the dynamic pressure feedback. This filter keeps the desirable effects of pressure feedback on shaping dynamic response, as well as eliminates the undesirable static error when in static state. It could be realized by electric circuit or software. The cut-off frequency is always chosen as 80%~90% of the system natural frequency.
According to Eq.(11), the cut-off frequency is:

$$f_{cutoff} = 0.85 \cdot f_N = 198 rad / s \tag{14}$$

So, the transfer function of high-pass filter is:

$$G_{DPF} = \frac{2\pi \cdot s / f_{cutoff}}{2\pi \cdot s / f_{cutoff} + 1} = \frac{0.032s}{0.032s + 1} \tag{15}$$

Fig.16 gives the new position response. The static and dynamic performances of EHA are both improved.

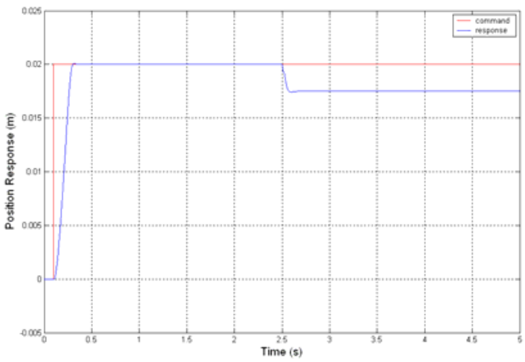


Figure 14 Step response of state feedback controlled EHA

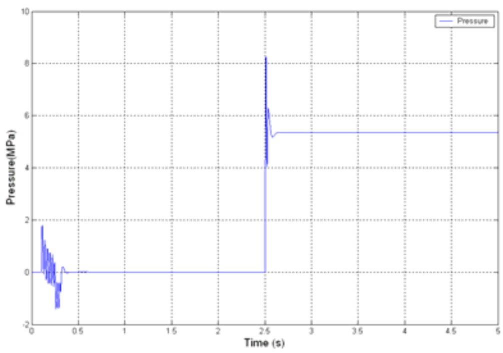


Figure 15 Pressure response of state feedback controlled EHA

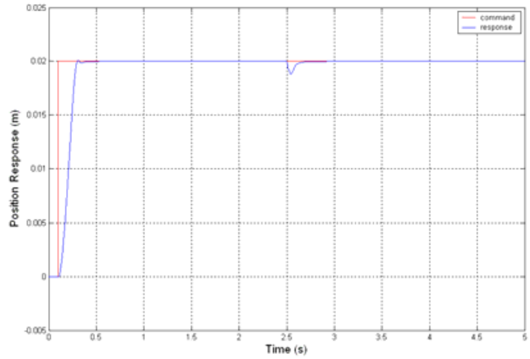


Figure 16 Step response of state feedback controlled EHA along with dynamic pressure feedback

## CONCLUSIONS

The block diagram model of FPVM-EHA is established in this paper. It is an accuracy model that contains more information than transfer function model.
The single PID, cascade PID and state feedback controllers are applied to this model for comparison analysis. The single PID control can't satisfy the EHA requirements; the cascade PID control efficiently helps the system to achieve high rapidity and stiffness, but leads to some oscillations; the state feedback control along with dynamic pressure feedback strategy is finally proven to be the best solution for improving both static and dynamic performance of EHA.

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