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STUDY OF OPTIMAL DESIGN AND LEG INERTIA EFFECT IN LARGE HYDRAULIC STEWART PLATFORM

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ABSTRACT

The optimum design of parallel manipulators is an important and challenging problem. Currently, much of optimization work has been done over several criteria related to workspace, stiffness, dexterity and conditioning index. Relatively few papers have taken the control problem into consideration. In this paper, an optimal design method based on generalized natural frequency is proposed, which aims to expand the bandwidth for the control of large hydraulic Stewart platform. A Lagrangian formulation which considers the whole leg inertia is presented to obtain accurate equivalent inertia matrix, based on which, the influence of design parameters on generalized natural frequency is studied. Conclusion drawn from numerical examples, based on more accurate model, demonstrates that the leg inertia especially the piston part plays an important role on the dynamics, and five design parameters (diameters of the moving platform and the base, piston mass, effective driving area and fully retracted leg length) influence the frequency most.

KEY WORDS

Large hydraulic Stewart platform, Optimal design, Control bandwidth, Lagrangian

NOMENCLATURE

A_1 :	effective area of the piston side	L_{oil2}:	equivalent chamber length of the rod side
A_2 :	effective area of the rod side	l_{pis} :	piston length
$\boldsymbol{A}_{du}$:	rotation matrix	l_i :	length between dli and the ith upper joint
$\boldsymbol{D}$:	Jacobian matrix	m_{pis}:	piston mass
d_i :	ith leg length	$\boldsymbol{M}_p$:	mass-inertia matrix of moving platform on $\sum O_p$
$\boldsymbol{F}$:	hydraulic driven force vector	$\boldsymbol{n}_{li}$:	unit vector along the ith leg
$\boldsymbol{f}$:	generalized natural frequency vector	P :	total potential energy
$\boldsymbol{F}_{ext}$:	external generalized force	R :	radius of the base
$\boldsymbol{I}_i$:	ith cylinder inertia of inertia matrix in $\sum O$	$\boldsymbol{R}_i$:	transformation matrix
$\boldsymbol{I}_i^{b_i}$:	ith cylinder mass moment about Bi in $\sum B_i$	$\boldsymbol{q}$:	$[x\ y\ z\ \psi\ \theta\ \varphi]^T$
$\boldsymbol{I}_{li}$:	ith piston inertia matrix	$\boldsymbol{v}_i$:	unit vector along the ith leg
K :	total kinetic energy	β :	oil bulk modulus
K_h :	stiffness of hydraulic oil spring	ρ :	density of the piston part
$\boldsymbol{K}_q$:	equivalent stiffness matrix	$\boldsymbol{\omega}_i$:	ith cylinder angular velocity
L_{oil1}:	equivalent chamber length of the piston side		

INTRODUCTION

In 1947 McGough proposed a six-degree-of-freedom platform, which was later used by Stewart (1965) in his flight simulator. In 1978 Hunt suggested using Stewart platform as manipulator.

In the research area of parallel manipulators, the optimum design is an important and challenging problem [1]. Two issues are involved in the optimum design: performance evaluation and synthesis. The latter is to determine the design parameters.

Parallel manipulators' performances depend heavily on their geometry. So much of the research work on parallel manipulators optimization has been done over several criteria related to the workspace [2, 3]. Other authors optimized the structural stiffness of the manipulator [4]. Also, some works may be referred where the optimization criteria used are related with the manipulability, dexterity, payload, conditioning index, or best accuracy . Different methods have been taken to solve the optimum design problems including the cost-function approach, interval analysis, and so on. Relatively few optimization works take the control problem into consideration. Shiller Z [5] used the motion time along the path as the optimization cost function. Khatib O [6] investigated the problem of manipulator design for increased dynamic performance which was characterized by the inertia and acceleration properties of the end-effector. However, the control of hydraulic actuators is more difficult than the control of electrical counterparts especially when manipulators are large.

The purpose of this optimization work is to expand the bandwidth for the control of large hydraulic Stewart platform based on generalized natural frequency. The rest of the paper is organized as follows. Section 2 gives the Lagrangian formulation which considers the whole leg inertia. The optimum method based on generalized natural frequency is introduced in Section 3. Numerical examples are finally carried out to validate and confirm the efficiency of the method in Section 4, in which the influence of design parameters on frequency is studied.

LAGRANGIAN FORMULATION

Several approaches [7] have been proposed for dynamic analysis of Stewart platform. This optimization work is to expand the bandwidth for the control based on generalized natural frequency, and the Lagrangian method is a direct way to get the equivalent mass matrix. To be accurate, the whole leg inertia should be considered. In this paper the legs are decomposed into two parts: the fixed part (to the base) and the moving part (the piston part). The integration method is used to calculate the energy of each part, with this method the energy of particle system includes all the translational and rotational energy.

Kinematics

The Stewart platform is shown in Figure 1.

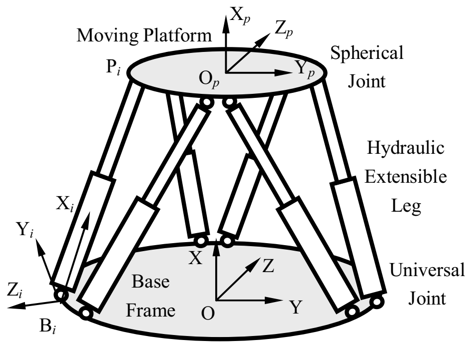

Figure 1 Hydraulic Gough-Stewart platform

The transformation matrix $\boldsymbol{R}_i$ from the leg coordinate to the base coordinate can be obtained as in [8].

Let the rotation matrix be defined by the roll, pitch, and yaw angles, namely, a rotation of φ about the x-axis, followed by a rotation of ψ about the y-axis, and a rotation of θ about the z-axis. Thus, it can be defined as

$$\boldsymbol{A}_{\mathrm{du}} = \boldsymbol{R}(y,\psi)\boldsymbol{R}(z,\theta)\boldsymbol{R}(x,\varphi) \tag{1}$$

The length of the ith leg is given by

$$d_i = f_i\left(x_{\mathrm{p}}, y_{\mathrm{p}}, z_{\mathrm{p}}, \psi, \theta, \varphi\right) \tag{2}$$

It yields

$$\dot{\boldsymbol{d}} = \boldsymbol{D}\dot{\boldsymbol{q}} \tag{3}$$

where $\boldsymbol{D}$ is the Jacobian matrix, $\dot{\boldsymbol{q}} = [\dot{x}_{\mathrm{p}}\ \dot{y}_{\mathrm{p}}\ \dot{z}_{\mathrm{p}}\ \dot{\psi}\ \dot{\theta}\ \dot{\varphi}]^{\mathrm{T}}$.

Piston part

The ith leg velocity vector can be written as

$$\begin{bmatrix} \dot{L}_{xi} \\ \dot{L}_{yi} \\ \dot{L}_{zi} \end{bmatrix} = \begin{bmatrix} \dot{x}_{\mathrm{p}} \\ \dot{y}_{\mathrm{p}} \\ \dot{z}_{\mathrm{p}} \end{bmatrix} + \dot{\boldsymbol{A}}_{\mathrm{du}} \begin{bmatrix} x_{\mathrm{u}i} \\ y_{\mathrm{u}i} \\ z_{\mathrm{u}i} \end{bmatrix} = \begin{bmatrix} \boldsymbol{J}_{xi} \\ \boldsymbol{J}_{yi} \\ \boldsymbol{J}_{zi} \end{bmatrix} \dot{\boldsymbol{q}} \tag{4}$$

where $\{x_p\, y_p\, z_p\}$ is the upper platform center coordinate in $\sum$O, and $\{x_{ui}\, y_{ui}\, z_{ui}\}$ is the ith upper joint coordinate in $\sum \mathrm{O_p}$.

In Figure 2, the coordinate of particle dl_i in $\sum$O is

$$\begin{bmatrix} x_{\mathrm{l}i} \\ y_{\mathrm{l}i} \\ z_{\mathrm{l}i} \end{bmatrix} = \frac{\left(d_i - l_i\right)}{d_i} \left(\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} - \begin{bmatrix} x_{\mathrm{d}i} \\ y_{\mathrm{d}i} \\ z_{\mathrm{d}i} \end{bmatrix} \right) + \begin{bmatrix} x_{\mathrm{d}i} \\ y_{\mathrm{d}i} \\ z_{\mathrm{d}i} \end{bmatrix} \tag{5}$$

where l_i is the length between dl_i and the ith upper joint, $\{x_{di}\, y_{di}\, z_{di}\}$ is the ith down joint coordinate in $\sum$O, and $\{x_i\, y_i\, z_i\}$ is the ith upper joint coordinate in $\sum$O.

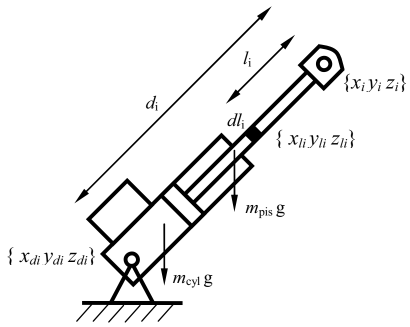

Figure 2 Leg of the Hydraulic Stewart platform

The kinematic energy of dl_i can be written as

$$dT_{1i} = \frac{1}{2}\rho dl_i v_{1i}^2 \tag{6}$$

where $\rho = m_{\mathrm{pis}} / l_{\mathrm{pis}}$, m_{pis} is the piston mass, and l_{pis} is the piston length.

Cylinder part

The velocity of the ith upper joint can be written as

$$\boldsymbol{v}_i = \dot{\boldsymbol{d}}_i \cdot \boldsymbol{n}_{1i} + \boldsymbol{\omega}_i \times \boldsymbol{d}_i \tag{7}$$

No rotation is allowed about the leg axis, so the angular velocity of the cylinder part can be written as

$$\boldsymbol{\omega}_i = \boldsymbol{n}_{1i} \times \boldsymbol{v}_i / \boldsymbol{d}_i \tag{8}$$

It yields

$$\boldsymbol{\omega}_i = \left(\begin{bmatrix} L_{yi}\boldsymbol{J}_{zi} - L_{zi}\boldsymbol{J}_{yi} \\ L_{zi}\boldsymbol{J}_{xi} - L_{xi}\boldsymbol{J}_{zi} \\ L_{xi}\boldsymbol{J}_{yi} - L_{yi}\boldsymbol{J}_{xi} \end{bmatrix} / d_i \right) \dot{\boldsymbol{q}} = \boldsymbol{J}_{\omega}\dot{\boldsymbol{q}} \tag{9}$$

Hence the total kinematic energy of the pistons is

$$\sum_{i=1}^{6} \frac{1}{2} \boldsymbol{\omega}_i^{\mathrm{T}} \boldsymbol{I}_i \boldsymbol{\omega}_i = \sum_{i=1}^{6} \frac{1}{2} \dot{\boldsymbol{q}}^{\mathrm{T}} \boldsymbol{J}_{\omega}^{\mathrm{T}} \boldsymbol{I}_i \boldsymbol{J}_{\omega} \dot{\boldsymbol{q}} \tag{10}$$

where $\boldsymbol{I}_i = \boldsymbol{R}_{\mathrm{i}} \boldsymbol{I}_i^{\mathrm{b}_i} \boldsymbol{R}_{\mathrm{i}}^{\mathrm{T}}$, $\boldsymbol{I}_i^{\mathrm{b}_i}$ is the mass moment inertia of the ith leg about Bi expressed in the leg coordinate.

The lagrangian dynamic formulation

With the principle of virtual work and Lagrange equation, the hydraulic driven force can be written as

$$\boldsymbol{F} = \boldsymbol{D}^{-\mathrm{T}} \left(\frac{d}{dt}\frac{\partial K}{\partial \dot{\boldsymbol{q}}} - \frac{\partial K}{\partial \boldsymbol{q}} + \frac{\partial P}{\partial \boldsymbol{q}} - \boldsymbol{F}_{\mathrm{ext}} \right) \tag{11}$$

where K is the total kinetic energy, P is the total potential energy, $\boldsymbol{F}_{ext}$ is the external generalized force.

THE OPTIMUM METHOD

Equivalent inertia matrix

The equivalent inertia matrix is

$$\boldsymbol{M}_{\mathrm{q}} = \boldsymbol{M}_{\mathrm{P}} + \sum_{i=1}^{6} \boldsymbol{I}_{1i} + \sum_{i=1}^{6} \boldsymbol{J}_{\omega}^{\mathrm{T}} \boldsymbol{I}_i \boldsymbol{J}_{\omega} \tag{12}$$

where

$$\boldsymbol{I}_{1i} = \frac{1}{3}\rho \frac{{l_{\mathrm{pis}}}^3}{d_i^4} \boldsymbol{D}(i,:)^{\mathrm{T}} \boldsymbol{D}(i,:) \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} + \frac{\rho \boldsymbol{J}_i}{d_i^2} \left(d_i^2 l_{\mathrm{pis}} - d_i {l_{\mathrm{pis}}}^2 + \frac{{l_{\mathrm{pis}}}^3}{3} \right) +$$

$$\frac{2\rho}{d_i^3} \left(\frac{1}{2} d_i {l_{\mathrm{pis}}}^2 - \frac{1}{3} {l_{\mathrm{pis}}}^3 \right) \boldsymbol{D}(i,:)^{\mathrm{T}} \left(\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}^{\mathrm{T}} - \begin{bmatrix} x_{\mathrm{d}i} \\ y_{\mathrm{d}i} \\ z_{\mathrm{d}i} \end{bmatrix}^{\mathrm{T}} \right) \begin{bmatrix} \boldsymbol{J}_{xi} \\ \boldsymbol{J}_{yi} \\ \boldsymbol{J}_{zi} \end{bmatrix} -$$

$$\frac{2\rho {l_{\mathrm{pis}}}^3}{3 d_i^4} \boldsymbol{D}(i,:)^{\mathrm{T}} \boldsymbol{D}(i,:) \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} x_{\mathrm{d}i} \\ y_{\mathrm{d}i} \\ z_{\mathrm{d}i} \end{bmatrix} + \frac{1}{3} \frac{\rho {l_{\mathrm{pis}}}^3}{d_i^4} R^2 \boldsymbol{D}(i,:)^{\mathrm{T}} \boldsymbol{D}(i,:) ,$$

$\boldsymbol{J}_i = \boldsymbol{J}_{xi}^{\mathrm{T}} \boldsymbol{J}_{xi} + \boldsymbol{J}_{yi}^{\mathrm{T}} \boldsymbol{J}_{yi} + \boldsymbol{J}_{zi}^{\mathrm{T}} \boldsymbol{J}_{zi}$,

$\boldsymbol{M}_{\mathrm{p}}$ is the mass-inertia matrix of moving platform in ∑O.

Generalized natural frequency

It is assumed that the mechanical part is rigid, and the hydraulic oil can be compressed. The stiffness of the hydraulic spring is defined as

$$k_{\mathrm{h}} = \beta \left[\frac{A_1}{L_{\mathrm{oil1}}} + \frac{A_2}{L_{\mathrm{oil2}}} \right] \tag{13}$$

$$\boldsymbol{K}_{\mathrm{h}} = \begin{bmatrix} k_{\mathrm{h}} & & \\ & \ddots & \\ & & k_{\mathrm{h}} \end{bmatrix} \tag{14}$$

where β is the oil bulk modulus, A_1 is the area of piston side, A_2 is the area of rod side, L_{oil1} and L_{oil2} are the two equivalent chamber lengths of the cylinder .

It can be obtained

$$\boldsymbol{K}_{\mathrm{q}} = \boldsymbol{D}^{\mathrm{T}} \boldsymbol{K}_{\mathrm{h}} \boldsymbol{D} \tag{15}$$

The generalized natural frequency on 6-DOF ($x\ y\ z\ \varphi\ \psi\ \theta$) is given by

$$f_i = \frac{1}{2\pi} \sqrt{\frac{\boldsymbol{K}_{\mathrm{q}}(i,i)}{\boldsymbol{M}_{\mathrm{q}}(i,i)}} \quad (i=1\sim6) \tag{16}$$

The optimization scheme

In applications with requirements of high precise positioning and good dynamic performance, e.g. large flight simulators, the control of the platform is complicated and difficult, especially for the hydraulic platform. In general, the control of hydraulic actuators is more challenging than that of their electrical counterparts when parallel manipulators are large. They exhibit a significant nonlinear behavior. The factors such as nonlinear flow/pressure characteristics, variations in the trapped fluid volume due to piston motion, fluid compressibility, flow forces and their effects on the spool position, and friction, all contributing to this nonlinear behavior. This will influence the actual control bandwidth, and it is less than half of the natural frequency in engineering. To expand the theoretical bandwidth for the control, the natural frequency characteristics must be considered in the optimal design.

For large hydraulic Stewart platform with requirements, the lowest natural frequency in the total workspace and the generalized natural frequency when all the actuators are at their mid stroke are the key frequencies. The aim of the design is to obtain highest frequencies, and the natural frequencies when all actuators are at their mid stroke should be as close as possible.

The optimization work is not based on the cost function. The steps in the optimization are as follows:

Step 1 Choose an initial set of design parameters. It can be roughly determined from the workspace requirement, the desired linear and angular isotropic accelerations at some velocity state.

Step 2 Determine the range of each design parameter, and give the graph results of the influence by the design parameters.

Step 3 Choose a new set of design parameters from step 2, get the task frequencies. If it's not satisfied, change the deign parameters, especially the effective hydraulic driving area and oil bulk modulus (the system oil should be preprocessed if necessary), return to the step 2.

Step 4 Compute the average hydraulic system power as a design reference by the system flow rate and pressure with design parameters.

Step 5 Workspace verifying and other requirements examination.

In the paper, the configuration is representative for a group of nearby or symmetric configurations. Based on the natural frequency, the bandwidth for the control will be determined more appropriate for the designer related to the control of the hydraulic parallel manipulator.

NUMERICAL EXAMPLE

Frequency verifying

The design parameters of the Stewart platform are shown in Table 1.

With the accurate inertia matrix, the generalized natural frequency of the Stewart platform can be obtained as in Eq. (16). With the mathematical model, the generalized natural frequency at the initial pose is:

[19.849 42.180 28.979 41.104 42.180 19.849] Hz

Table 1 Parameters of the Stewart platform

Parameter	Value
Upper Diameter	2.1 (m)
Down Diameter	5.4 (m)
Upper joints angle	21 (°)
Down joints angle	21 (°)
Initial height	2.5 (m)
Length of the piston	2.3 (m)
Length of the cylinder	2.2 (m)
Mass of the moving platform	300 (Kg)
Mass of the piston	200 (Kg)
Mass of the cylinder	350 (Kg)

An ADAMS model is built to validate the mathematical model, and the corresponding frequency is:

[19.848 42.181 28.978 41.103 42.181 19.848] Hz

The effect of leg inertia on the Stewart platform

Comparisons between current model and traditional one are made on the static and dynamic driving forces.

In the simulation of static forces, all the velocities and accelerations remain zero, there is no the external force and torque exerted on the platform, and the moving platform moves horizontally along z-axis between -250mm and 250mm. It's obvious that the inertia of the leg especially the piston part influences a lot on the static driving forces.

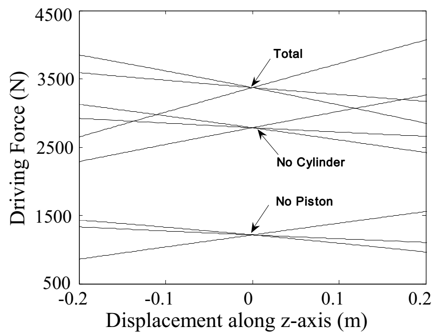

Figure 3 Static driving forces with models considering the whole leg or part of the leg

In figure 4, the moving platform moves horizontally along z-axis with a sinusoidal motion (100sin(πt) mm), while other velocities and accelerations remain zero. The three lines (*N_1, N_2, N_3*) are the results with current inertia matrix model, and the other three lines (*O_1, O_2, O_3*) are with traditional one including only the translational part of the leg.

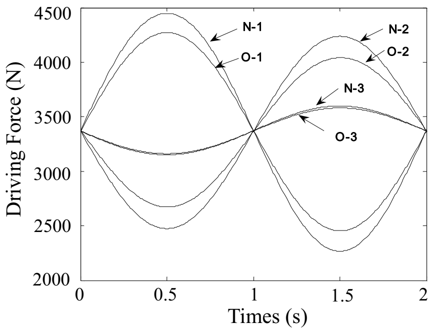

Figure 4 Dynamic driving forces

Influence of design parameters on natural frequency

The influence of design parameters on natural frequency is shown in figure 5-figure 13. Curve 1, curve 2, curve 3, and curve 4 represent the six frequencies when all the actuators are at their mid stroke, and curve 5 represents the lowest frequency in workspace.

With the figures, the influence of parameters can be easily observed. Five design parameters (figure 5 7 9 11 12) influence the frequency most. Especially in figure 7, the frequency lines are highly non-linear, the influence by the down diameter is complicated, which should be paid more attention during the design. If necessary, the oil should be preprocessed, which is effective for increasing the frequencies simultaneously.

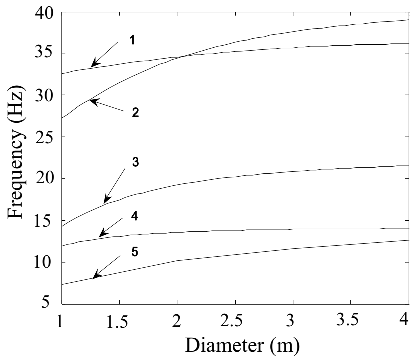

Figure 5 Influence of upper diameter

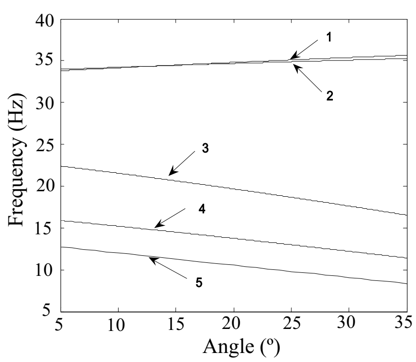

Figure 6 Influence of upper joints angle

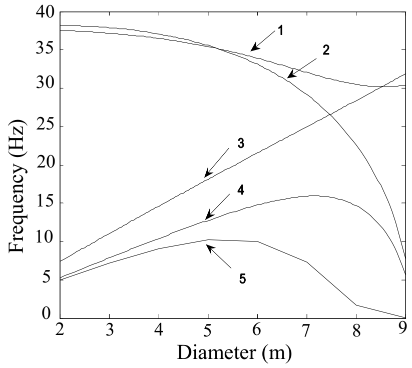

Figure 7 Influence of down diameter

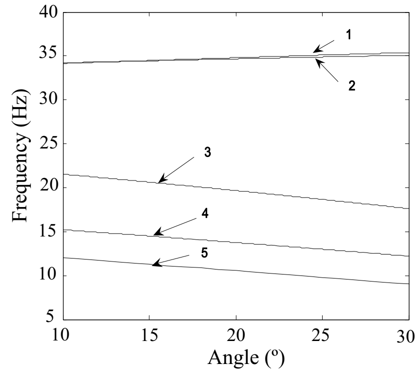

Figure 8 Influence of down joints angle

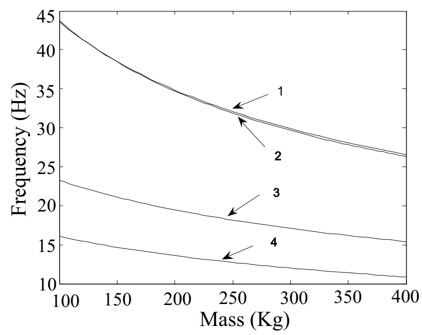

Figure 9 Influence of piston mass

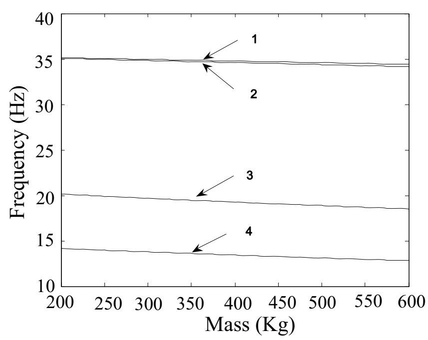

Figure 10 Influence of cylinder mass

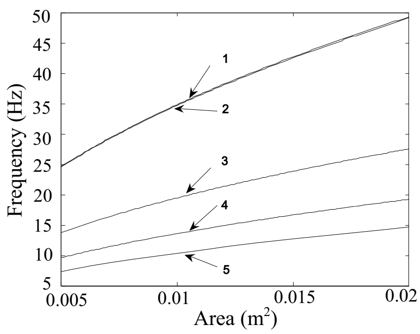

Figure 11 Influence of driving area

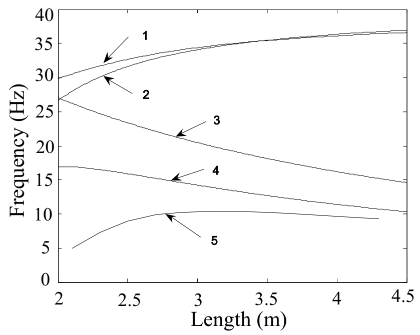

Figure 12 Influence of fully retracted leg length

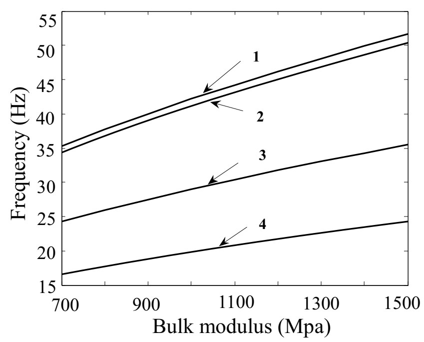

Figure 13 Influence of oil bulk modulus

CONCLUSIONS

This paper presented an optimal design method based on generalized natural frequency, which aims to expand the bandwidth for the control of large hydraulic Stewart platform. The ADAMS model validates and confirms the efficiency of the current model. And numerical examples were carried out and came to the following conclusions:

(1) Current model is more accurate than the traditional one.
(2) The leg inertia especially the piston part plays an important role in the dynamics.
(3) Five design parameters (diameters of the moving platform and the base, piston mass, effective driving area and fully retracted leg length) influence the frequency most.

This optimization method can be used with other requirements. In later work, it will be combined with hydraulic system design (flow rate/pressure, flow power and cooling power). It's an efficiency method to obtain a compact hydraulic Stewart platform with higher bandwidth. It will provide a set of appropriate design parameters for the large hydraulic Stewart platforms. It is suitable for the optimum design of other hydraulic parallel manipulators with higher bandwidth.

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