3C2-3

Proceedings of the 6th JFPS International
Symposium on Fluid Power, TSUKUBA 2005
November 7-10, 2005

A NEW MASS FLOW-RATE MODEL OF GAS THROUGH ORIFICE-TYPE RESTRICTORS IN AEROSTATIC BEARINGS

Jyh-Chyang RENN and Jia-Feng YANG

Department of Mechanical Engineering,
National Yunlin University of Science and Technology
Douliu, Yunlin, Taiwan 640, R.O.C.
(E-mail:rennjc@yuntech.edu.tw)

ABSTRACT

This paper proposes a new mathematical model to describe the mass flow-rate characteristics of gas through orifice-type restrictors in aerostatic bearings. In the conventional design of gas-lubricated aerostatic bearings, the mass flow-rate of gas through an orifice-type restrictor is generally derived from a well-known mathematical model, which is originally developed to describe the mass flow-rate property of gas through an ideal nozzle. In this paper, however, a series of CFD-simulations and experiments according to ISO 6358 are carried out and the results show that the mass flow-rate characteristics through an orifice-type restrictor are different from those through an ideal nozzle. Consequently, the conventional model to determine the mass flow-rate of gas through orifice-type restrictors in aerostatic bearings may have to be modified and updated to the proposed new model for more precise design and modeling of the gas-lubricated aerostatic bearings.

KEY WORDS

Mass Flow-rate, CFD, Aerostatic Bearing, Orifice, Restrictor

NOMENCLATURE

A	: area of the restrictor
b	: critical pressure ratio
C	: the sonic conductance
C_0	: orifice discharge coefficient
g	: acceleration due to gravity
k	: specific heat ratio of gas
$\dot{m}$	: mass flow-rate
P_1	: absolute pressure at inlet
P_2	: absolute pressure at outlet
R	: gas constant
T_1	: absolute temperature at inlet
T_2	: absolute temperature at outlet
ρ_2	: density of gas at outlet
Ψ	: coefficient of flow through orifice
ϕ	: test restrictor's diameter

INTRODUCTION

Aerostatic bearings have been widely used for measuring instruments, machine tools, dental drills, jet engines and computer peripheral devices because of their substantially low friction loss and heat generation. Like other fluid lubricated bearings, aerostatic bearings serve two purposes. One is to support an external load and the other is to lubricate a pair of surface. Nowadays, the aerostatic bearings are commonly used in the field of precision engineering. Figure 1 shows the schematic sectional view of an aerostatic bearing with an orifice-type restrictor. The gas from an external source is fed into the clearance space through flow restrictors which are often orifice-type feed holes in one of the bearing surface, and escapes continuously to the atmosphere from the outside edges of the bearing. However, the mass flow-rate of the gas through such an orifice throat is generally derived from a well-known mathematical model, which is originally developed to describe the mass flow-rate of gas through an ideal nozzle. It is reasonable to doubt if there is any difference between the property of mass flow-rate of gas through an orifice and that through a nozzle.

To simplify the analysis, we propose a new model and assume initially that the mass flow-rate property through an orifice is similar to that through an ideal nozzle. Thus, the formulas of the latter to calculate the mass flow-rate are preserved. The basic idea of this paper is trying to tune the parameters used in the formulas such that the formulas for the ideal nozzle may also be applied to the case of the orifice. Figure 2 indicates such a mass flow-rate property through an ideal nozzle, in which the constant maximal mass flow-rate of the ideal gas is noticeable. Later in this paper, it will be shown that this maximal value depends on a parameter C, which is generally called the sonic conductance. Besides, if the ratio of the outlet pressure to the inlet pressure is small, meaning that the pressure drop across the nozzle is quite large, the gas-flow through the nozzle becomes a jet with high velocity. The acoustic velocity, however, physically limits this velocity through the nozzle. This is precisely the reason why the constant maximal flow-rate exists in the flow property shown in Fig. 2.

The parameter *b*, called the critical pressure ratio, is defined as the intersection of the straight flow-rate of the so-called choked flow and the elliptic flow-rate curve of subsonic flow. For an ideal nozzle with a fixed cross-sectional area, the parameters C and b are constants because the former depends chiefly on the cross-sectional area and the latter is proved analytically to be a constant value of 0.5283. Similarly, for a fixed orifice, it is also reasonable to assume that both the parameters C and b are constants. The remaining question is how to derive the best-fit values for parameters C and b. In this paper, therefore, the CFD simulations as well as a series of experiments according to ISO 6358 are introduced to determine the best values for the two parameters.

Surveying some previous reports [1-3, 4, 6], it is found that the model to calculate the mass flow-rate of gas through orifice-type restrictors copies without exception the conventional model originally developed for ideal nozzles. One interesting paper presented by the author dealt with the experimental approach to build a new mass flow-rate model for gas through orifice-type restrictors [7]. However, the dimension of the test restrictor is too large as compared with that of an actual orifice-type restrictor installed in industrial aerostatic bearings.

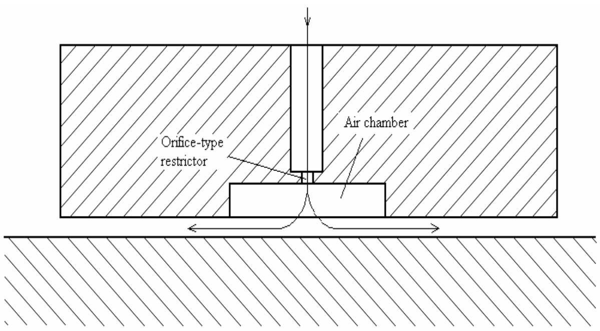

Figure 1 Orifice-type restrictor in aerostatic bearings

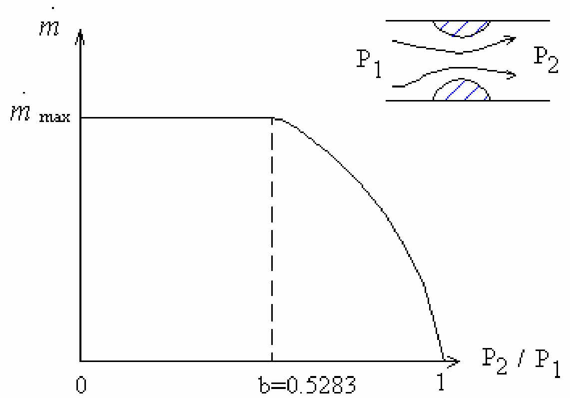

Figure 2 The mass flow-rate property through an ideal nozzle

MODEL OF THE MASS FLOW-RATE CHARACTERISTICS THROUGH AN IDEAL NOZZLE

The mass flow-rate of gas through a restrictor can be described by the generalized formulas [5, 7].

$$\dot{m} = C\rho_0 P_1 \sqrt{\frac{T_0}{T_1}} \qquad for\ \frac{P_2}{P_1} \le b\,, \tag{1}$$

and

$$\dot{m} = C\rho_0 P_1 \sqrt{\frac{T_0}{T_1}} \sqrt{1-\left(\frac{P_2/P_1-b}{1-b}\right)^2}\ for\ \frac{P_2}{P_1} > b \tag{2}$$

Where $\dot{m}$: mass flow-rate

C : the sonic conductance,

b : critical pressure ratio,

P_1 : absolute pressure at inlet,

P_2 : absolute pressure at outlet,

T_1 : temperature at inlet,

T_0 : outlet temperature under standard reference conditions (293.15K),

ρ_0 : density under standard reference conditions (1.189 kg/m^3).

If the restrictor is an ideal nozzle, the critical pressure ratio is given by

$$b = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}, \tag{3}$$

where k denotes the specific heat ratio of the gas.

On the other hand, similar equations, which are commonly used in the modeling of the mass flow-rate of gas through an orifice-type restrictor in aerostatic bearings, are summarized as [1, 2, 3, 4, 6]

$$\dot{m} = AC_0 \Psi \frac{P_1}{g\sqrt{RT_1}}\,, \tag{4}$$

$$\Psi = \sqrt{2g\frac{k}{k-1}\left(\frac{2}{k+1}\right)^{\frac{2}{k-1}}}\ \text{ for }\ \frac{P_2}{P_1} \le \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}, \tag{5}$$

$$\Psi = \sqrt{2g\frac{k}{k-1}\left\{\left(\frac{P_2}{P_1}\right)^{\frac{2}{k}} - \left(\frac{P_2}{P_1}\right)^{\frac{k+1}{k}}\right\}}\ \text{ for }\ \frac{P_2}{P_1} > \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}, \tag{6}$$

where A: area of the restrictor,

C_0 : orifice discharge coefficient,

R : gas constant,

Ψ: coefficient of flow through orifice,

g : acceleration due to gravity.

The Eq. (5) and Eq. (6) represent the sonic and subsonic flow conditions respectively. The orifice discharge coefficient, C_0, is basically a contraction coefficient Generally, it is derived experimentally.

Comparing Eq. (1) with Eqs. (4)-(5), the sonic conductance can be rewritten as

$$C = \frac{AC_0}{g\rho_0}\sqrt{\frac{2gk}{(k-1)RT_0}\left(\frac{2}{k+1}\right)^{\frac{2}{k-1}}}\ . \tag{7}$$

Hence, the sonic conductance depends chiefly on the area of the restrictor. Obviously, the set of Eqs. (1)-(3) is nearly equivalent to the set of Eqs. (4)-(6) if the temperature from inlet to outlet keeps unchanged. One minor difference, however, is the functions used to describe the subsonic mass flow-rate property. One is the simplified Eq. (2) and the other is the original Eq. (6). The maximal deviation, however, is only 3% [5].

For ideal gas, k = 1.4. According to Eq. (3), the value of critical pressure ratio is found to be 0.5283. However, we propose that the value of *b* may not equal 0.5283 for other types of restrictor. Consequently, the conventional model described by Eqs. (4)-(6) to calculate the mass flow-rate of gas through an orifice-type restrictor may have to be modified because such a model is originally developed only for the nozzle-type restrictor.

In this paper, the Eqs. (1), (2) and (7) are recommended for the analysis of the mass flow-rate properties through orifice-type restrictors in aerostatic bearings because they are much easier for the numerical calculation. In the following, both the values of the parameters *C* and *b* are determined by simulation as well as experimental approach.

CFD-SIMULATION OF THE ORIFICE-TYPE RESTRICTOR

In this paper, the commercial software package CFD-RC is utilized as a tool to analyze the properties of mass flow-rate through orifice-type restrictors. The test probe with a built-in orifice-type restrictor is shown in Fig. 3. Moreover, four different diameters (ϕ =0.1 mm, 0.1445mm, 0.182 mm and 0.25 mm) of the test restrictor are chosen for the simulation. Figure 4 indicates the schematic geometry of the test restrictor with diameter ϕ =0.1 mm.

In the development of the mass flow-rate equations for nozzles, it is generally accepted that nozzles of different cross-sectional areas possess the same mass flow-rate properties described by Eqs. (1)-(3) or Eqs. (4)-(6). In addition, the critical pressure ratio given by Eq. (3) is a constant (*b*=0.5283) for nozzles of different cross-sectional areas and the sonic conductance given by Eq. (7) is proportional to the cross-sectional area of the nozzle. Similarly, it is proposed in this study that orifices of the same structure but of different cross-sectional areas should have the same air flow properties described by the parameters *C* and *b*. Moreover, the value of *b* should be a constant for orifices of different cross-sectional areas and the value of *C* should be proportional to the cross-sectional area of the orifice.

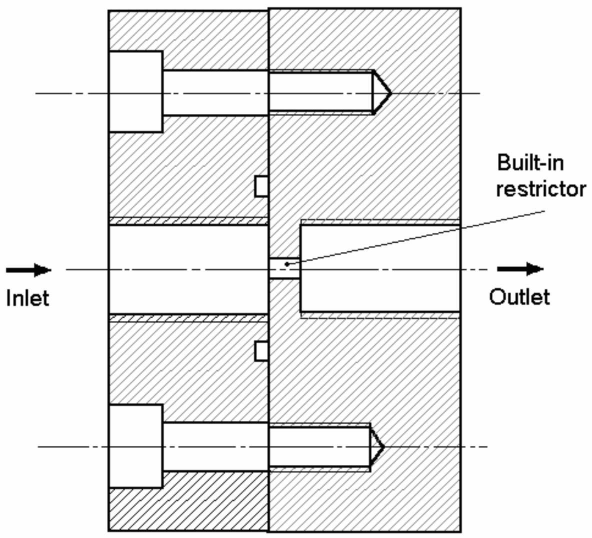

Figure 3 Test probe with built-in test restrictor

In the following, the CFD simulation is employed to numerically determine the mass flow-rate of gas through the test restrictor. Since the gas-flow in the test restrictor is axis-symmetric, a two-dimensional Cartesian coordinate model of the flow field is established for the quantitative analysis of the mass flow-rate through the test restrictor (ϕ =0.1 mm) as shown in Fig. 5.

The initial and boundary conditions for the CFD-simulation are:

(1) The input velocity of the gas-flow is set to be 36 m/s. Different setting of input velocity influences only the speed of convergence of the numerical calculation. It, however, does not affect the accuracy of numerical analysis.

(2) The gas flow is compressible.

(3) The gas density is assumed to be $1.189 Kg/m^3$ at room temperature.

(4) The dynamic viscosity of the gas is set to be $1.789 \times 10^{-5}\, N \cdot s / m^2$ at room temperature.

(5) The room temperature is set to be 20 $^\circ C$ or 293 $^\circ K$.

(6) The reference acoustic velocity of gas is assumed to be 340 m/s.

(7) The boundaries of the flow field are considered as the wall, which means that no gas-flow across the boundaries is allowed.

(8) No heat transfer and chemical reaction exists in the flow field.

(9) The outlet conditions are set to be atmospheric pressure and room temperature.

After CFD simulations, the velocity vector diagram of the test restrictor is found and shown in Fig. 6. From this velocity vector diagram and the corresponding numerical velocity data at the outlet, the average mass flow-rate of the gas can be derived. Other relevant simulation results together with the experimental data are discussed in the following section.

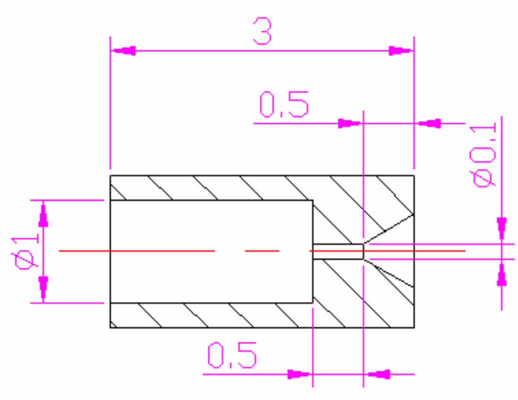

Figure 4 Geometry of the restrictor (ϕ =0.1 mm)

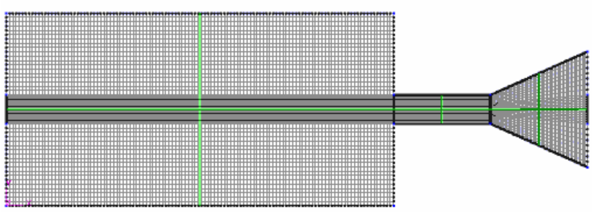

Figure 5 Two-dimensional Cartesian coordinate model of the flow field (ϕ =0.1 mm)

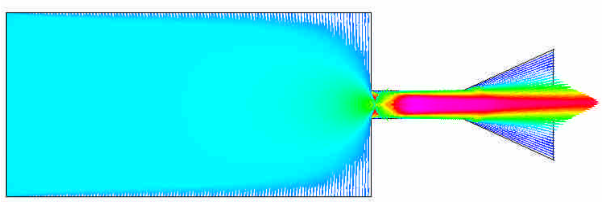

Figure 6 The velocity vector diagram (ϕ =0.1 mm)

EXPERIMENTAL RESULTS AND DISCUSSION

Figure 7 shows the schema of test bench for the orifice-type test restrictor according to ISO 6358. The most important device is the mass flow meter, which serves to measure the mass flow-rate of gas through the test restrictor. At the inlet to the test restrictor, a pressure regulator is employed to vary the supply pressure. Moreover, two pressure sensors are utilized in this test bench. One is installed at the inlet and the other at the outlet. The measured pressure signals are used to calculate the pressure ratio of the outlet pressure to the inlet pressure. All the measured data are sampled and stored in the PC by using a data-acquisition software.

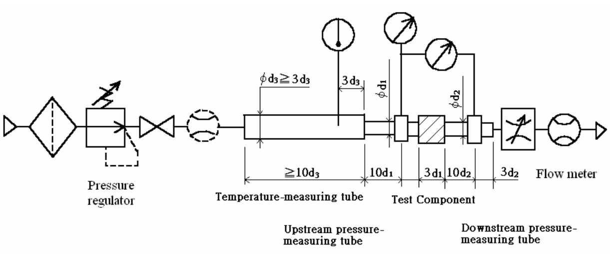

Figure 7 Test bench for the orifice-type restrictor according to ISO 6358

In this study, the method of exhaust-to-atmosphere is utilized. The outlet pressure, P_2, is basically the atmospheric pressure plus the negligible pressure drop across the mass flow meter (the flow control valve is kept fully open). Hence it is approximately a constant. To vary the pressure ratio P_2/P_1, only the inlet pressure, P_1, is adjusted by using the pressure regulator. In a real aerostatic bearing, however, the inlet pressure is kept constant and the outlet pressure is variable according to different external loads. Obviously, these two conditions are basically equivalent since only the pressure ratio P_2/P_1 is employed in the calculations of the mass flow-rate through the orifice-type restrictor. Besides, the outlet pressure for the orifice-type restrictor in an aerostatic bearing is defined as the pressure in the air-chamber at the outlet as shown in Fig. 1.

Figure 8 shows the simulation curves as well as experimental results of the mass flow-rate of gas through test restrictors with different diameters. Obviously, every curve shown in Fig. 8 can be divided into two parts. At higher supply pressure, the curve is more like a straight line. On the contrary, at lower supply pressure, the curve can be described by an elliptical function. The intersection of these two parts is exactly the critical pressure ratio. Figure 9 indicates the values of the critical pressure ratio as a function of the test restrictor's diameter. It is observed that the critical pressure ratio for an orifice-type restrictor is approximately a constant value of 0.36.

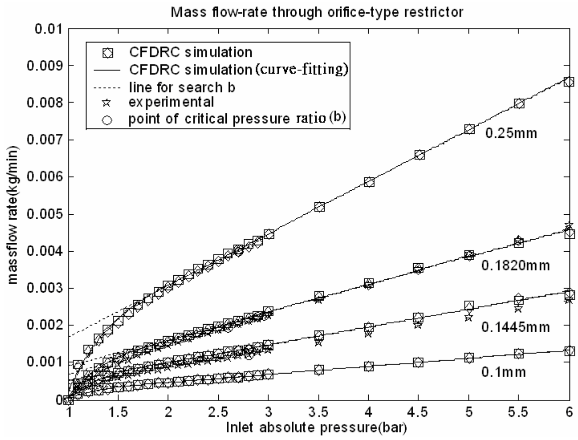

Figure 8 Simulation and experimental results of the mass flow-rate as a function of different diameters

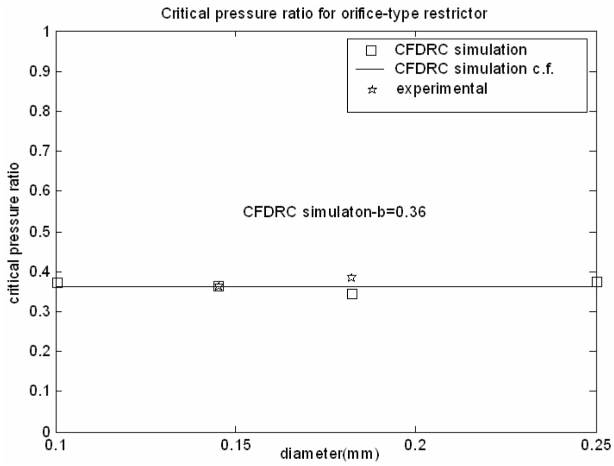

Figure 9 Critical pressure ratio as a function of different diameters

Figure 10 shows the graphical relation between the sonic conductance and the pressure ratio for different test restrictor's diameters. It is observed that only two experimental curves corresponding to test restrictors of diameters 0.1445 mm and 0.182 mm are shown in Fig. 10. This is chiefly because the test restrictors of diameters 0.1 mm and 0.25 mm are not available. It is further noticeable that every curve in Fig. 10 is quite similar to the curve of mass flow-rate through an ideal nozzle shown in Fig. 2 except the value of the critical pressure ratio. This proves one of our previous assumptions that the mass flow-rate property through an orifice is similar to that through an ideal nozzle. Figure 10 further illustrates the sonic conductance is approximately proportional to the square of the test restrictor's diameter. Such a proportional relation proves the second assumption that the sonic conductance depends chiefly on the cross-sectional area of the orifice-type restrictor.

Finally, the orifice discharge coefficient C_0 can be determined from Eq. (7) and Fig. 10. After some calculations, C_0 is found to be 0.83, which agrees very well with the general assumption used in the conventional model.

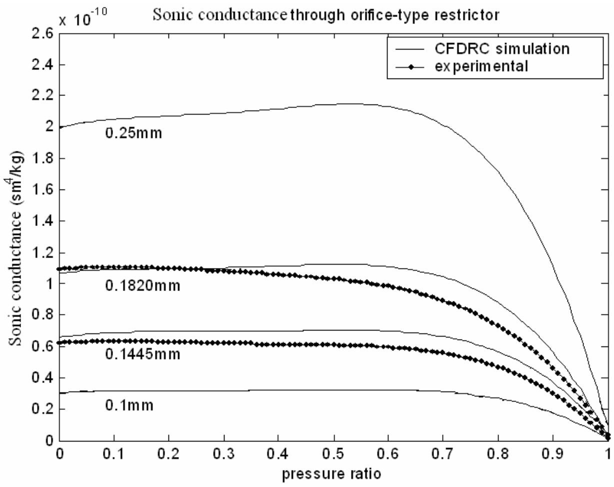

Figure 10 Sonic conductance as a function of pressure ratio for different test restrictor's diameters

CONCLUSION

In this paper, we have proposed a new model for the calculation of the mass flow-rate of gas through orifice-type restrictors in the aerostatic bearings. Four conclusions may be drawn from this research.

(1) The experimental results agree very well with the results of CFD simulation.

(2) The simulation and experimental results have both shown that the major difference between the conventional and the new proposed model is the value of the critical pressure ratio. In details, for an orifice-type restrictor of which the length is larger than the diameter, the recommended value for the critical pressure ratio is in the range $0.35<b<0.4$ instead of 0.5283.

(3) The orifice discharge coefficient shown in Eq. (7) is found experimentally to be 0.83. Hence, the recommended value for the orifice discharge coefficient is in the range $0.8 <C_0<0.85$. This value is applicable only to the orifice-type restrictor of which the length is larger than the diameter.

(4) The proposed new model, which is described by Eqs. (1), (2) and (7), is recommended for the determination of the mass flow-rate of gas through an orifice-type restrictor in aerostatic bearings.

ACKNOWLEDGMENT

The financial supports of the *National Science Council* under grant number NSC-92-2212-E-224-005 is greatly appreciated.

REFERENCES

1. Powell J.W., Design of aerostatic bearing, Machinery Publishing Co. Ltd., 1970.
2. Kogure K, Kaneko R, Ohtani K., "A study on characteristics of surface restriction compensated gas bearing with T-shaped grooves," Bulletin, JSME 1982, Vol. 25, No. 210, pp. 2039-2045.
3. Nakamura T. and Yoshimoto S., "Static tilt characteristics of aerostatic rectangular double-compound restrictors," Tribology International 1996, Vol. 29, No. 2, pp. 145-152.
4. Lin G., Tojiro A. and Ichiro I., "A computer simulation method for dynamic and stability analysis of air bearing," Wear 1998, Vol. 126, pp. 307-319.
5. Murrenhoff H., Grundlagen der Fluidtechnik, Teil 2 : Pneumatik, Umdruck zur Vorlesung, 1. Auflage, RWTH Aachen, Germany, 1999.
6. Chen M.F. and Lin Y.T., "Static behaviour and dynamic stability analysis of grooved rectangular aerostatic thrust bearings by modified resistance network method," Tribology International 2002, Vol. 35, p. 329-338.
7. Renn J.C. and Hsiao C.H., " Experimental and CFD study on the mass flow-rate characteristics of gas through orifice-type restrictor in aerostatic bearings," Tribology International 2004, Vol. 37, pp. 309-315.