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SIMULATION OF DYNAMICS PROCESSES OF HYDRAULIC SYSTEM WITH AXIAL-RECIPROCATING PISTON PUMP AND ELECTRICAL ENGINE

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ABSTRACT

In the article the dynamic processes in an axial-reciprocating hydraulic pump and asynchronous engine are considered. The mathematical model of an axial-reciprocating hydraulic pump is presented, where the flow of fluid in each cylinder of the pump and the interaction of liquid with the piston are taken into account. The flow of fluid in a hydraulic system is described by a system of equations of a hyperbolic type, which is solved by the characteristics method. For example, the mathematical simulation of the activity of an axial-reciprocating pump in a hydraulic system together with the mechanical drive is shown.

KEY WORDS

axial-reciprocating hydraulic pump, dynamics, the interaction, numerical method

NOMENCLATURE

a_x :	acceleration to x direction;
c :	speed of sound in a liquid;
d :	diameter of cylinder (pipeline);
e :	wall thickness of the pipeline;
E :	modulus elasticity of a material of the pipeline;
F :	force;
$\{F_{fr}\}$:	vector of friction moment;
$F_1(x)$:	discharge of liquid mass to the unit of the length in a pipeline;
$F_2(x)$:	kinetic energy of the liquid flow in a pipeline to the unit of area;
f :	coefficient of pressure losses along pipe $f = f(Re)$;
$f_1(v,p)$:	functions dependent on velocity and pressure;
$f_2(v,p)$:	functions dependent on velocity and pressure;
h_0 :	constructive parameter of a pump,
I_e :	moments of inertia of mass;
I_{LP} :	set of number low pressure cylinder;
i :	number of iteration;

$[J]$: Jacobi matrix;
$K(p)$: bulk modulus elasticity of liquid;
L_r: completely reduced rotor inductivity;
L_S: completely reduced stator inductivity;
L_μ: magnetization circuit inductivity;
M_e: electrical engine resistance moment;
M_{fr}: friction moment;
M_r: resistance torque;
m: mass of cylinder;
$m_{inp}(x)$: mass flow of a liquid in a unit area;
NZ: number of pistons in a pump;
p: liquid pressure;
p_D: liquid pressure at a point D;
p_F: liquid pressure at a point F;
p_{Gi}: liquid pressure i on the surface of piston;
p_L: liquid pressure at a point L;
p_R: liquid pressure at a point R;
pol: number of pole pairs;
r_s: active resistance of stator;
r_r: active resistance of rotor;
q_G: displacement of the piston at a point G;
q_H: displacement of the piston at a point H;
$\dot{q}_G \equiv \frac{dq}{dt}$: speed of the piston at a point G;
$\dot{q}_H \equiv \frac{dq}{dt}$: velocity of the piston at a point H;
Re: Reynolds number;
$S(x)$: cross-section area of the pipeline;
S_i: cross-sectional area of cylinder number i;
S_S: cross-sectional area of the piston;
v: liquid velocity;
v_D: liquid velocity at a point D;
v_F: liquid velocity at a point F;
v_L: liquid velocity at a point L;
v_R: liquid velocity at a point R;
$v_{inp}(x)$: velocity of a liquid acting in the pipe line;
x_F: x coordinate of pint F;
$\{Y\}$: vector of unknowns;
γ: isentropic exponent;
ε: relative volume of gas in the liquid;
$\{\Phi\}$: vector of algebraic equations;
φ_e: turn angle of rotor of electrical engine;
φ_i: turn angle of the cylinder number i;
τ: integration time step of differential equations;
$\Pi(x)$: perimeter of cross section of the pipeline;
ρ: liquid density;
σ: shearing stresses on the internal surface of the pipe line;
ν: kinematic viscosity of a liquid;
$\{\psi\}$: the bound flow vector.

INTRODUCTION

The main research object in the present work is axial-reciprocating pump which is intended for work as power units in different guidance schemes. The researches of axial-reciprocating pump were carried out by many authors. But many of them considered the system of hydraulic pump as the system with lumped values and thus did not take into consideration wave processes going on in hydraulic lines. The purpose of present work is to show the wave processes going as in hydraulic pump system, using the characteristic method. The axial-reciprocating pump is analyzed together with asynchronous engine, higher and lower pressure pipelines. Movement of liquid is accepted as one-dimensional and unsettled, i.e. all local velocity are considered. The liquid movement is considered as one-dimensional, i.e. all local velocity are equal to average velocity, and unsettled. Velocity and pressure depend on longitude coordinate and time. Such liquid movement is characterized by the wave of increased and reduced pressure which spreads from the place of change in each pressure vibration cross-section and in deformation of pipeline walls. The liquid is double-component mixture, including gas (air). The presence of air in liquid sharply decreased sonic velocity in liquid.

MATHEMATICAL MODEL OF HYDRAULIC AXIAL-RECIPROCATING PUMP

Movement and continuity equations of viscous, compressible fluid in pressure pipe have the following form

$$\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{fv|v|}{2d} + a_x = 0;$$

$$\frac{\partial p}{\partial t} + v\frac{\partial p}{\partial x} + c^2\rho\frac{\partial v}{\partial x} = 0, \qquad (1)$$

Let research hydraulic system consisting of the electrical asynchronous engine (AE), axial-reciprocating pump, pipelines, hydraulic resistance, hydraulic valve and tank. The scheme of axial-reciprocating pump is shown in Fig. 1. The dynamic model of asynchronous engine and axial reciprocating piston pump is studied as a complicated hydraulic and mechanical system.

One of the most progressive variants of automated electrical driver is an alternating current electrical driver with asynchronous engine. The foundation of the AE mathematical model consists of differential equations of electrical and mechanical balance and equations of electromagnetic energy transformation to mechanical energy.

To estimate dynamic regimes of AE, two-phase mathematical models are used . In the general case AE two-phase model consists of the system of differential and algebraic equations:

$$\{\dot{\psi}\} = \lfloor A_\psi \rfloor \{\psi\} + \{B_\psi(t, \psi, \dot{\varphi}_e)\} \tag{2}$$

$$I_e \frac{d^2 \varphi_e}{dt^2} = M_e(\psi) - M_r(\varphi_i, \dot{\varphi}_i, p_{Gi}), \tag{3}$$

where matrix $\lfloor A_\psi \rfloor$ and vector $\{B_\psi(t, \psi, \dot{\varphi}_e)\}$ are equal

$$[A_\psi] = a_\psi \begin{bmatrix} -r_s L_r & 0 & r_s L_\mu & 0 \\ 0 & -r_s L_r & 0 & r_s L_\mu \\ r_r L_\mu & 0 & -r_r L_s & 0 \\ 0 & r_r L_\mu & 0 & -r_r L_s \end{bmatrix},$$

$$\{B_\psi(t, \psi, \dot{\varphi}_e)\} = \begin{Bmatrix} \sqrt{2} U_{nom} \cos(w_s t) \\ -\sqrt{2} U_{nom} \sin(w_s t) \\ \dot{\varphi}_e \psi_4 \\ -\dot{\varphi}_e \psi_3 \end{Bmatrix},$$

and torque of AE is equal

$$M_e = \frac{3}{2} \cdot pol \cdot L_\mu a_\psi (\psi_1 \psi_4 - \psi_2 \psi_3), \tag{4}$$

$$a_\psi = \frac{1}{L_s L_r - L_\mu^2}.$$

The engine resistance moment is equal

$$M_r(\varphi_i, \dot{\varphi}_i, p_{Gi}) = \sum_{i=1}^{NZ} S_i h_0 \cdot p_{Gi} \sin \varphi_i + M_{fr}(\dot{\varphi}_i), \tag{5}$$

where i -th cylinder turning angle is equal

$$\varphi_i = \varphi_e + (i-1) 2\pi / NZ .$$

The dynamic model of axial-reciprocating pump is constructed by evaluating the principle of action and constructive peculiarities of the pump (Fig. 1).

Every piston of a pump moves in a cylinder and closely interacts with liquid. Depending on the rotor turning angle φ_e, every piston of a pump ejects (high pressure) and sucks-in (low pressure) liquid. During the suction and at the insufficient liquid discharge, zones of cavitation form on the surface of a piston.

The interaction between the separate piston and a pump and liquid is studied.

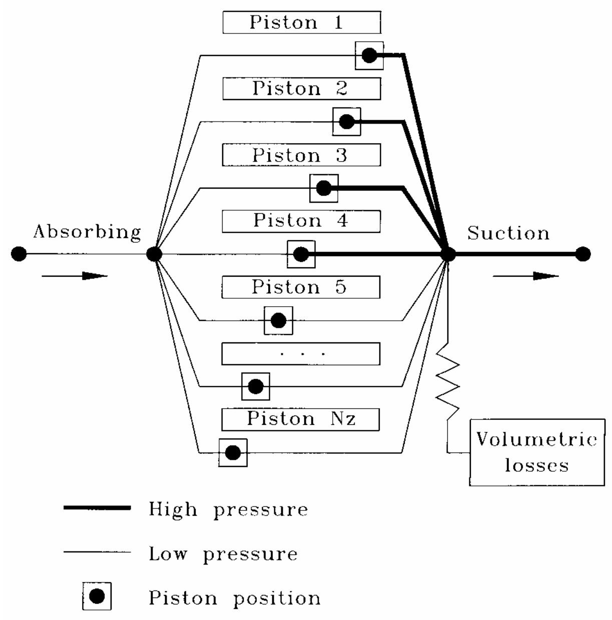


Figure 1 The hydraulic circuit of axial reciprocating piston pump

According to the construction of an axial reciprocating piston pump, every piston is coupled with rotor; therefore interaction of piston and liquid flow in a cylinder shall be studied together with equations of electrical engine movement (1-5).

The presented dynamic model of axial reciprocating piston pump evaluates: local and volumetric losses of pressure in every cylinder; occurrence of cavitation zones, identification of their variable volume; quantity of gas in liquid; transient processes in an electrical engine; various constructional parameters of a pump; influence of a hydraulic driver on dynamic processes of a pump; change of cross-section area when transferring from the suction to a pressure pipeline and vice versus. Differential equations of liquid movement in the cylinder are solved by characteristics method [1,2]. The main idea of characteristics method is the fact that unknown variable speed and liquid pressure at instant moment of time $t + \tau$ is determined according to these parameters at a moment of time (Fig. 2). Pressure and velocity in point D at the moment of time is determined from nonlinear algebraic equation system

$$C^+ : \Phi_1 = v_D - v_L + \frac{1}{2}(p_D - p_L)\left[\left(\frac{1}{\rho c}\right)_L + \left(\frac{1}{\rho c}\right)_D\right] - \frac{\tau}{2}\left[\left(\frac{f_1}{\rho c}\right)_L + \left(\frac{f_1}{\rho c}\right)_D\right] - \frac{\tau}{2}\left[(f_2)_L + (f_2)_D\right] = 0; \qquad (6)$$

$$C^- : \Phi_2 = v_D - v_R - \frac{1}{2}(p_D - p_R)\left[\left(\frac{1}{\rho c}\right)_R + \left(\frac{1}{\rho c}\right)_D\right] + \frac{\tau}{2}\left[\left(\frac{f_1}{\rho c}\right)_R + \left(\frac{f_1}{\rho c}\right)_D\right] - \frac{\tau}{2}\left[(f_2)_R + (f_2)_D\right] = 0 \qquad (7)$$

$$f_1(v,p) = \frac{c_2}{S(x)}\left(F_1 - \rho v \frac{\partial S}{\partial x}\right);$$

$$f_2(v,p) = \frac{1}{\rho S(x)}(F_2 - vF_1) - \frac{\sigma \Pi(x)}{\rho S(x)} - a_x ,$$

$$F_1 = \Pi(x) m_{inp}(x) v_{inp}(x), \; F_2 = \Pi(x) m_{imp}(x) v_{inp}(x),$$

$$\sigma = \frac{\rho f(\mathrm{Re}) v |v|}{8},$$

$$f(\mathrm{Re}) = \begin{cases} \dfrac{75}{\mathrm{Re}}, \textit{when} \quad \mathrm{Re} \le 2320; \\ \dfrac{0{,}31464}{\mathrm{Re}^{0{,}25}}, \textit{when} \quad \mathrm{Re} > 2320 \end{cases}$$

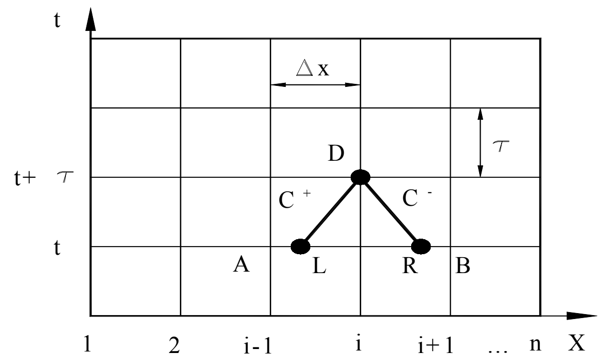


Figure 2 Scheme of point parameters determination by characteristic method

The velocity of sound in a two-phase liquid is placed in pipeline as determined:

$$c = \sqrt{\frac{K(p)/\rho}{1 + \frac{K(p)\cdot d}{E \cdot e} + \frac{\varepsilon}{\gamma}\left[\frac{K(p)}{\gamma p} - 1\right]}}, \qquad (8)$$

The system of equations (6) and (7) is solved by a Newton method:

$$[J]_i \{Y\}_i = -\{\Phi\}_i , \qquad (9)$$

$$\{Y\}^T = [p_D, v_D]; \; \{\Phi\}^T = [\Phi_1, \Phi_2].$$

The kinetic energy of the rotor of pump is transformed to potential energy of a liquid in a high-pressure cylinder.
For accuracy simulation of interaction of the hydraulic pump piston with liquid four cases of interaction are considered. Each case of this interaction depends on a rule of the piston in the cylinder and on computational grid. At the moment of interaction of a liquid with the piston the volumetric losses of the liquid are taken into account [4,5].
The system, which consists of the body moving in a pipeline with liquid (Fig. 3).

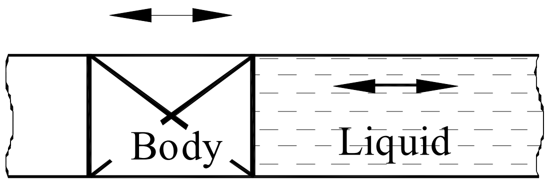


Figure 3 The circuit of the system "body, liquid, and pipeline"

The parameters of liquid flow and body interaction in a discrete pipeline are determined in four cases. In this article is presented only first case of interaction between piston and liquid flow.
The first case (Fig. 4).
In this case the following conditions shall be fulfilled [5]:

$$x_{i-1} \le q_H < x_i , x_{i-1} < q_H + \dot{q}_H \tau < x_i ,$$

$$x_{i-1} < q_H + (v_H + a_H)\tau < x_i .$$

The liquid flow parameters of point F are interpolated among points H, C and B. The final system of equation, from which parameters of liquid flow and body movement of point G can be determined, is:

$$\Phi_1 = q_G - x_F - [v_F - a(p_F)] = 0 ,$$

$$\Phi_2 = v_G - v_F - \frac{1}{2}(p_G - p_F)\left[\left(\frac{1}{\rho a}\right)_F + \left(\frac{1}{\rho a}\right)_G\right] - \frac{\tau}{2}\left[\left(\frac{f_1}{\rho a}\right)_F + \left(\frac{f_1}{\rho a}\right)_G\right] - \frac{\tau}{2}\left[(f_2)_F + (f_2)_G\right] = 0$$

$$\Phi_3 = S_S\left[\frac{2}{\tau}(q_G - q_H) - \dot{q}_H\right] - S_C v_G - fout\sqrt{|p_G - p_{out}|}\cdot sign(p_G - p_{out}) = 0\ ,$$

$$\Phi_4 = m\left[\frac{4}{\tau^2}(q_G - q_H) - \frac{4}{\tau}\dot{q}_H - \ddot{q}_H\right] - S_S p_G + F(t, q_G) = 0\ . \qquad (10)$$

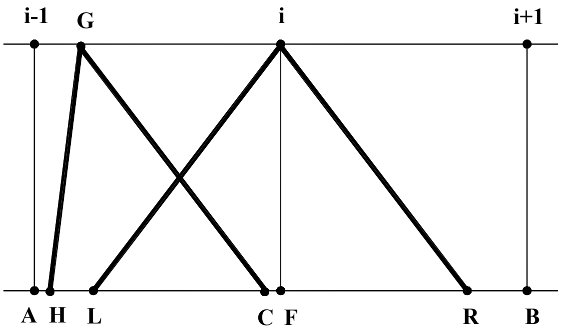
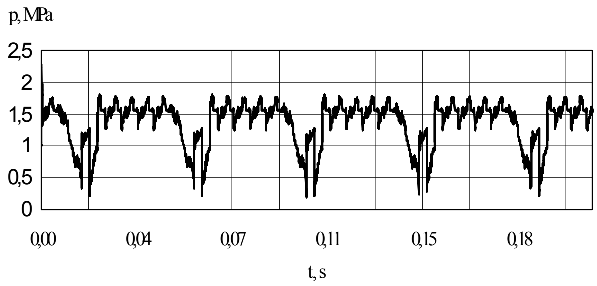


Figure 4 Cases of body and liquid flow interaction

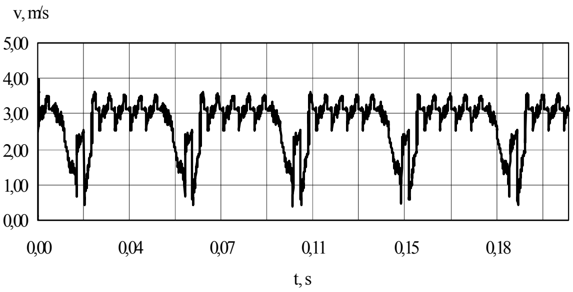
Parameters of liquid flow of point i are determined from the system of equations (6) and (7).

COMPUTATIONAL RESULTS

As an example of the axial reciprocating piston pump 2G15-14 is considered, the shaft of which is turned by asynchronous engine 4A132S4Y3. Pulsations of liquid flow pressure and velocity at the output of axial reciprocating piston pump when the quantity of gas in liquid is equal to 0.1% zero ($\varepsilon = 0.001$) are shown in Fig. 5.



a



b

Figure 5. Change of pressure and velocity in time at the output of axial reciprocating piston pump, when the quantity of gas in liquid is equal to 0.1% zero ($\varepsilon = 0.001$):a – pressure, b – velocity

CONCLUSIONS

- The composed mathematical model of a axial-reciprocating hydraulic pump takes into account wave motion of a liquid, local losses of pressure and volumetric fluid leakages, phenomenon of a cavitation, gaps in the mechanical drive
- The designed technique is possible for using in research volumetric hydraulic transmissions

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