# COMPARISON OF LINEAR CONTROLLERS FOR A HYDRAULIC SERVO SYSTEM

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# ABSTRACT

In many hydraulic control applications, classic linear controllers are still employed, although there exist a number of number of nonlinear control methods, which may be better suited for handling the intrensic non-linearities often found in hydraulic systems. The focus of this paper is therefore on comparing different linear controllers, based on both simulation and experimental results, to determine what is obtainable when applying standard linear controllers to a hydraulic SISO servo system. The paper furthermore addresses how the performance may be improved by using internal pressure control and model based information to include feedforward information. The control strategies considered are all based on measurement of piston position and pressure only.

## **KEYWORDS**

Pressure control, feedforward control, controller comparison

### NOMENCLATURE

$A_p$	:	Cylinder piston area
$B_t$	:	Viscous friction coefficient
$C_L$	:	Leakage coefficient
$K_q$	:	Linearised flow coefficient
$\hat{K_{qp}}$	:	Linearised flow/pressure coefficients
$K_v$	:	Servo valve coefficient
$M_{eq}$	:	Equivalent mass working on cylinder
$p_L, p_S, t$	$p_T$ :	Load, supply and tank pressure
u	:	Control signal
$V_t$	:	Total cylinder chambers volume
$x_p, x_r$	:	Cylinder and reference cylinder position
ÂE		Effective oil bulk modulus

## INTRODUCTION

Generally, hydraulic plants exhibit significant nonlinearities and have time-varying parameters, which makes them difficult to control using linear controllers, as these typically have to be designed rather conservatively in order to ensure stability. Still, linear controllers are often applied to hydraulic systems [1], despite many non-linear control algorithms have been applied to hydraulic systems with success, see e.g. [1]-[4]. The main reasons for this are the simplicity of the linear control theory and the well developed set of rules and methods that exist, combined with the relative low knowledge of non-linear control theory within the hydraulic industri. Therefore, this paper focus on what performance, in terms of tracking errors and robustness, that is obtainable using standard linear controllers on a hydraulic servo system. It also considers what may be obtained if these are also combined with model based feedforward information. The paper furthermore addresses how the damping may be increased by adding an internal pressure feedback. Based on both simulation and experimental results the controllers are compared and their possibilities and limitations are outlined.

## SYSTEM PRESENTATION

The hydraulic application considered here is a two-d.o.f. robotic manipulator, where each link is driven by a servo valve controlled symmetrical cylinder. Each of these servo valve and cylinder systems represents a hydraulic servo system (HSS) as considered in the following. A generalized model of such a system is characterized by the highly non-linear nature of the servo valve pressure/flow characteristic, friction effects, a very low damping ratio and dynamics that strongly depends on the operating point and the physical parameters describing the system. If the non-linear equations governing this system are linearised around an operating point, the transfer function, relating cylinder piston position to the servo valve input signal, may be written as [5]:

$$G(s) = \frac{X_P(s)}{U(s)} = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \frac{1}{s}$$
(1)

where the gain K, natural frequency  $\omega_n$  and damping ratio  $\zeta$  are given by:

$$K = \frac{k_q A_P}{B_t (C_L + k_{qp}) + A_P^2}$$
(2)

$$\omega_n = 2\sqrt{\frac{\beta_F}{M_{eq}V_t}} (B_t(C_L + k_{qp}) + A_P^2) \tag{3}$$

$$\zeta = \frac{4\beta_F M_{eq}(C_L + k_{qp}) + B_t V_t}{4\sqrt{M_{eq} V_t \beta_F (B_t (C_L + k_{qp}) + A_P^2)}}$$
(4)

#### **Pressure Feedback**

In order to increase the damping in the system an internal pressure feedback loop is implemented, as this has a stabilising effect on the HSS. The general idea is to measure the pressure difference in the servo-cylinder and feeding it back to the load flow reference of the servovalve. However, instead of using a simple gain feedback, corresponding to a leakage term, the pressure is fed back through a high-pass filter which means, that only the high frequency pressure fluctuations will affect the load flow. A block diagram of the system with the pressure feedback loop inserted is shown in Figure 1.

The effect of the pressure feedback may be seen by closing the inner pressure loop, whereby the integrator part



Figure 1 Block diagram of the system with and without pressure feedback through a high-pass filter.

 $\frac{4\beta_F}{V_{\star s}}$  in the original system is changed to:

$$\frac{4\beta_F}{(4\beta_F K_{hp} + V_t)s} \cdot \frac{\tau_{hp}s + 1}{\frac{V_t}{4\beta_F K_{t-} + V_t}\tau_{hp}s + 1} \tag{5}$$

The first term in Eq. (5) can be seen to reduce the gain of the system and the second term in Eq. (5) is a lead filter, which, if dimensioned correctly, may be used to increase the relative stability of the system. Implementing this filter, by choosing the time constant,  $\tau_{hp}$ , below the natural frequency of the system and adjusting the gain,  $K_{hp}$ , to obtain sufficient phase margin and bandwidth, the systems frequency response may be changed as shown in the bode plot in Figure 2.



Figure 2 Bode plot of HSS to be controlled.

With the pressure control implemented, it shows that the resonance magnitude peak has been smoothened. This means, that the gain may be increased without encountering stability problems.

## LINEAR CONTROLLERS

Linear controllers are typically designed based on worst case considerations, in order to ensure stability in the whole work space. The model given by Eq. (1) is based on the cylinder being in the middle position and under the assumption that this is worst-case concerning stability, which is also the case for a constant inertia load. For the robotic manipulator the inertia load changes as a function of piston position, why the above is an approximation. The lowest eigenfrequency for each of the two cylinders are however so close to the middle position that this approximation is justified. This model is therefore used for determining the controllers and the controller parameters are adjusted, so the system fulfils the phase- and gainmargins requirements:

$$GM > 6[dB] \qquad PM > 45^o \tag{6}$$

In the following the linear controllers considered are shortly described. These controllers are all found interesting due to the Bode plot characteristic of the system shown in Figure 2.

- **P-controller:** As the system both with and without the pressure control is a type one system, a proportional feedback results in tracking, why this is the simplest and most obvious controller to try.
- **PT-/P-lead-controller:** From the system without pressure feedback the bode plot shows a fast decrease of the phase around the eigenfrequency, why adding a first order filter (PT) with a filterfrequency around  $\omega_{\Pi}$  may reduce the magnitude around the peak, but only change  $\omega_{\Pi}$  a little. The controller is:

$$G_{PT}(s) = K_P \frac{1}{\tau_i s + 1} \tag{7}$$

With the pressure control implemented it may be seen that adding phase to the system around the resonance frequency will increase the bandwidth of the system. Therefore a P-Lead controller is designed for this case:

$$G_{PD}(s) = K_P\left(\frac{\tau_d s + 1}{\beta \tau_d s + 1}\right) \tag{8}$$

• **PID-controller:** Since the system is of type one, the above controllers gives zero steady-state error, when the HSS is given a step input. If a PI or a PID controller is used, zero steady-state error will also be obtained for a ramp input. Practically the PID-controller is implemented as a PI-lead controller, in order to avoid problems with differentiation of high frequency noise:

$$G_{PID}(s) = K_P \left(1 + \frac{1}{\tau_i s}\right) \left(\frac{\tau_d s + 1}{\beta \tau_d s + 1}\right) \quad (9)$$

## **Simulation Results**

The responses of the different controllers with and without pressure feedback are shown in Figure 3 and 4. The reference for both cylinders are sinusoidal, making each of the cylinders move from one end to the other and back in 3 seconds.



Figure 3 Simulated position tracking errors for the sinusoidal reference input, using the three controllers without pressure feedback.



Figure 4 Simulated position tracking errors for the three controllers with pressure feedback.

Comparing the two figures it may be seen that the pressure feedback reduces the position error significantly. Still the errors are large, and typically not within limits for most of these types of servo systems. From the simulation results it may also be seen that the performance of the two servo systems are quite different, due to the different masses on the systems. Of the considered controllers it so also clear that the PID-controllers show the best results, and the PID-controller with pressure feedback is therefore the one tested on the laboratory setup.

#### **Experimental Results**

Originally the PID-controller was designed using a leadfilter for the D-part. When implemented in the laboratory the controller works, but the operation is noisy. In order to improve performance a critically damped second order filter in combination with the D-part reduces the noise significantly without reducing the performance, which implemented yields:

$$G_{PID}(s) = K_p + \frac{K_{pt}}{\tau_i s} + \frac{K_{pd}s}{\tau_d^2 s^2 + 2\tau_d s + 1}$$
(10)

Simulation results for the new PID controller with pressure feedback are shown in Figure 5, which also shows the robustness of the controller to a mass step of  $50 \ [kg]$ applied after 6 [s]. Experimental results for this controller are shown in Figure 6. The results corresponds very well with the simulated ones.



Figure 5 Simulated position tracking errors, using the PID controller with pressure feedback. A mass step of 50[kg] is applied after 6 [s].



Figure 6 Measured tracking errors on the laboratory setup, using the PID controller with pressure feedback.

## CONTROLLERS WITH FEEDFORWARD INFORMATION

The above controllers had a stabilising effect on the system. However when fast position tracking is required the controllers shows drawbacks such as phase lag, steadystate error and overshoot. Combining a standard controller with e.g. a velocity feedforward term may improve performance in an often simple and intuitive matter, assuming the necessary system information is available. Hereby the linear controller only needs to compensate for the error between the estimated and the correct control signal, whereas the feedforward terms ensures the main part of tracking. Estimation of the feedforward control signal may in this case be done based on the reference trajectory. Alternative approaches may be to include feedforward terms to eliminate or reduce influence of measurable disturbances. In case of the robotic manipulator this could be measuring of the acceleration of link 2, as the inertia forces of this affects link 1. A brief discussion of different approaches for including feedforward terms in the controller structure may be found in [3].

#### Velocity Feedforward Controller (VFC)

To apply this method we need  $x_r \in C^1$ , which is also the case for the sinusoidal input. The general idea is then to continuously compute the servo-valve control input from knowledge of the system. Since, in steady state, the velocity of the piston is (assumed) proportional to the displacement flow, and hereby the flow from the servo-valve, the spool position corresponding to the wanted flow can be calculated from the orifice equation and fed forward, as shown in Figure 7.



Figure 7 System with position feedback (proportional controller) and velocity feedforward control.

An expression for the load flow, corresponding to the average flow across the servo-valve orifices is given by:

$$Q_L = K_v u \sqrt{\frac{p_S - p_T - \operatorname{sign}(u)p_L}{2}}$$
(11)

Similarly, the load flow may be related to the pressure dynamics as:

$$Q_L = A_P \dot{x}_P + \frac{V_t}{4\beta_F} \dot{p}_L + C_L p_L \tag{12}$$

Which, if neglecting leakage reduces to:

$$Q_L = A_P \dot{x}_P + G \tag{13}$$

where G representing the pressure dynamics. Substitut-

ing Eq. (13) into Eq. (11) yields

$$u = \Gamma \dot{x}_P + \nu$$

$$\Gamma = \frac{A_P}{K_v \sqrt{\frac{p_S - p_T - \operatorname{sign}(u)p_L}{2}}}$$

$$\nu = \frac{g(p_L)}{K_v \sqrt{\frac{p_S - p_T - \operatorname{sign}(u)p_L}{2}}}$$
(14)

with  $\Gamma$  and  $\nu$  in general being state dependent. With Eq. (14) representing the system dynamics, the control law takes the following form.

$$u = \hat{\Gamma}(\dot{x}_R + Con(x_R - x_P)) + \hat{\nu}$$
(15)

where  $\hat{\Gamma}$  may be either estimated or calculated by measuring the load pressure  $p_L$ , based on Eq. (14). In the following  $\nu$  is set to zero. Con denotes the (position) feedback controller implemented,  $x_R$  is the position reference,  $x_P$  is the actual position and  $\dot{x}_R$  is the velocity reference. If  $\Gamma$  is assumed constant, the feedforward term acts as a pre-filter and does therefore not affect system stability. If  $p_L$  instead is fed back and used for calculating  $\Gamma$  the system changes and becomes state dependent. The control law implemented is thus given by:

$$u = \frac{A_P}{\underbrace{K_v \sqrt{\frac{p_S - p_T - \operatorname{sign}(u)p_L}{2}}}_{u_{feedforward}}} \dot{x}_R + \underbrace{Con(x_R - x_P)}_{u_{feedback}}$$
(16)

Three different types of the VFC have been tested on this basis, these are:

- VFCE with position measurement only (Estimated  $\Gamma$ )
- VFCA with position and load pressure measurement (Actual Γ)
- VFCP with pressure feedback (VFCA with pressure feedback though a high-pass filter)

All three types were tested with a PI controller in the (position) feedback loop.

### **Simulation Results**

Simulation results for the three different controllers are shown in Figure 8-10.

In all the simulations a mass step of 50 [kg] is applied after 6 [s]. From the simulations it may be seen that the best result is obtained when  $\Gamma$  is calculated using measurement of  $p_L$  (VFCA), but without the pressure feedback, as this actually may increase the tracking error, when these are small, i.e. in the range of 0.5 - 1 [mm]. For the VFCP adjustment of the controller parameters resulted in the integral term being so small that the controller is implemented as a pure proportional controller instead.



Figure 8 Simulated position tracking errors, using the VFCE and a PI-controller in the outer loop.



Figure 9 Simulated tracking errors, using the VFCA in combination with a PI-controller.

#### **Experimental Results**

All VFC strategies were tested in the laboratory to verify the simulation findings. The results of the measurements are shown in figs. 11-13.

Practical implementation showed that, due to noise, it was necessary to filter the measured  $p_L$  in the VFCA-scheme, in order to get stable and satisfactory performance. The filtering is done using a first order filter, however this did degrade performance, as the tracking error becomes much larger than in the simulation, as shown in Figure 12. For the VFCP-scheme the experiments showed that it was possible to implement the scheme without filtering the pressure signal, but the tracking error is still larger than what the simulation shows, cf. Figure 13 compared to Figure 9.



Figure 10 Simulated tracking errors, using the VFCP and a P-controller in the outer loop.



Figure 11 Measured tracking errors using the VFCE and a PI-controller in the outer position loop.



Figure 12 Measured tracking errors using the VFCA, with filtered  $p_L$ , and with a PI-controller in the position loop.

#### CONCLUSIONS

In this paper different linear controllers have been tested and compared, to determine what is obtainable when applying these to a highly non-linear hydraulic system. Based on both simulation and experimental results it was found that the linear controllers in their standard form are not generally sufficient to obtain acceptable performance. Including the pressure feedback dramatically improved performance. Of the standard controllers considered the PID-controller with pressure feedback shows the best response and also proved to be robust towards mass variation. In the practical implementation of the controller it was however necessary to include a filter on the derivative part to avoid noise problems. A disadvantage with this controller is as well that both a position and pressure transducer is required.

Considering the feedforward controllers, these all work well and have tracking errors better than for the PIDcontroller with pressure feedback. From both the experiments and simulation it was found that the best overall result is obtained with the simple VFCE strategy, using a PI-controller in the position feedback loop.



Figure 13 Measured tracking errors using the VFCP and a P-controller.

This although the VFCP in both simulation and experimentally shows very similar or better tracking errors. The advantages of the VFCE strategy is, besides being easy to implement, that it only requires a position measurement and it is not very sensitive to noise. The robustness towards mass variations on the other hand is not as good as for the PID-controller and the VCFA-scheme, which both requires pressure measurement, but still better than for the VFCP. Of the controllers considered the VFCA is the most robust to mass variations.

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