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FEEDBACK LINEARISATION APPLIED ON A HYDRAULIC SERVO SYSTEM

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ABSTRACT

Generally most hydraulic systems are intrensically non-linear, why applying linear control techniques typically results in conservatively dimensioned controllers to obtain stable performance. Non-linear control techniques have the potential of overcoming these problems, and in this paper the focus is on developing and applying several different feedback linearisation (FL) controllers to the individual servo actuators in a hydraulically driven servo robot to evaluate and compare their possiblities and limitations. This is done based on both simulation and experimental results.

KEYWORDS

Feedback linearisation, hydraulic servo system.

NOMENCLATURE

A_p	:	Cylinder piston area
B_t	:	Viscous friction coefficient
C_L	:	Leakage coefficient
F_L	:	External load force on the piston
K_v	:	Valve discharge coeffient
M_{eq}	:	Equivalent mass being accelerated
p_A, p_B	:	Cylinder chamber pressures
p_L	:	Load pressure, defined as $p_L = p_A - p_B$
p_S	:	Supply pressure
p_T	:	Tank pressure
u	:	Control signal to servo valve
x_P	:	Cylinder piston position
V_A, V_B	:	Volume in cylinder chambers
β_F	:	Effective oil bulk modulus

INTRODUCTION

In the last decades electrical drives have become increasingly popular, due to the advances in power electronics and frequency inverters, but hydraulic servo-systems still find a variety of applications in industrial motion control due to its high size-to-torque ratio [1]-[4]. The use of hydraulics is for instance still widespread in areas of machining plants, mining etc. [5]. Often however, these hydraulic drives are controlled using linear controllers, which degrade the obtainable performance of the drive, as most hydraulic systems are intrinsically non-linear and have time-varying parameters. The non-linearities, combined with large parameter ranges means that is difficult to achieve satisfactory performance, as the linear controllers have to be dimensioned conservatively to ensure stability. In addition, the natural damping in these system are in general very low. Non-linear control techniques have the potential of overcoming these problems, and in this paper, the problem of tracking control of electro-hydraulic servo actuators using feedback linearisation applied to a hydraulically driven test robot is addressed. In its simplest form, feedback linearisation in this way amounts to cancelling the non-linearities in a non-linear system, so that the closed-loop dynamics become linear and may hence be controlled by linear controllers.

SYSTEM DESCRIPTION

The system considered in this paper is a two degrees-of-freedom rotary arm manipulator with a high-frequency servo valve controlled hydraulic cylinder driving each link. An illustration of the system is shown in Figure 1. This system is, as many hydraulic systems, characterised by the highly non-linear nature of the servo valve pressure/flow characteristic, friction effects, a very low damping ratio and dynamics that strongly depends on the operating point and the physical parameters describing the system.

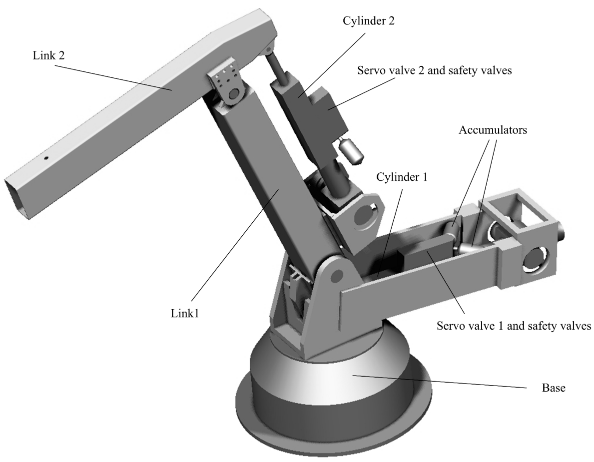

Figure 1 3D illustration of the hydraulic servo robot used as test case.

Considering each of the servo valve controlled cylinders separately, these may be model by the following equations. The force balance equation applied to the cylinder piston is given by:

$$M_{eq}\ddot{x}_P = p_L A_P - B_t \dot{x}_P - F_L \tag{1}$$

Neglecting the dynamics of the servo valve, the flow through the servo valve may be described by:

$$Q_A = \begin{cases} K_v u \text{sign}(p_S - p_A)\sqrt{|p_S - p_A|} & , u \geq 0 \\ K_v u \text{sign}(p_S - p_B)\sqrt{|p_S - p_B|} & , u < 0 \end{cases}$$
$$Q_B = \begin{cases} K_v u \text{sign}(p_B - p_T)\sqrt{|p_B - p_T|} & , u \geq 0 \\ K_v u \text{sign}(p_A - p_T)\sqrt{|p_A - p_T|} & , u < 0 \end{cases} \tag{2}$$

The cylinder chamber pressures are found from the continuity equation:

$$\dot{p}_A = \frac{\beta_F}{V_{A,0} + A_P x_P}(Q_A - A_P \dot{x}_P - C_L p_L) \tag{3}$$

$$\dot{p}_B = \frac{\beta_F}{V_{B,0} - A_P x_P}(A_P \dot{x}_P - Q_B + C_L p_L) \tag{4}$$

INPUT-OUTPUT LINEARISATION OF THE HSS MODEL

The above model is the basis for the input-output linearisation. Representing this as a state model, with the state vector $\underline{x} = [x_P \;\; \dot{x}_P \;\; p_A \;\; p_B]^T$:

$$\dot{\underline{x}} = \begin{bmatrix} x_2 \\ \frac{1}{M_{eq}}(A_P(x_3 - x_4) - B_t x_2 - F_L) \\ \frac{\beta_F}{V_A}(-A_P x_2 - C_L(x_3 - x_4)) \\ \frac{\beta_F}{V_B}(A_P x_2 + C_L(x_3 - x_4)) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{\beta_F}{V_A} H(x_3, x_4) \\ -\frac{\beta_F}{V_B} I(x_3, x_4) \end{bmatrix} u \tag{5}$$

where $H(x_3, x_4) = Q_A$ and $I(x_3, x_4) = Q_B$ are the flow expressions given by Eq. (2).

Compensation of Pressure Dynamics

In the following the feedback linearisation approach will be used to compensate for non-linearities related to the pressure dynamics. This is done by considering only the part of the model, with the state vector $\underline{x}^T = [p_A \;\; p_B]$:

$$\dot{\underline{x}} = \begin{bmatrix} \frac{\beta_F}{V_A}(-A_P x_2 - C_L(x_3 - x_4)) \\ \frac{\beta_F}{V_B}(A_P x_2 + C_L(x_3 - x_4)) \end{bmatrix} + \begin{bmatrix} \frac{\beta_F}{V_A} H(x_3, x_4) \\ -\frac{\beta_F}{V_B} I(x_3, x_4) \end{bmatrix} u \tag{6}$$

Define the output as the force:

$$y = A_P(p_A - p_B) = A_P(x_3 - x_4) \tag{7}$$

Following the approach in [6], we set up the quantities:

$$\nabla h = [A_P \;\; -A_P] \tag{8}$$

$$L_g h = \nabla h \underline{g} \tag{9}$$
$$= \frac{\beta_F A_P}{V_A} H(x_3, x_4) + \frac{\beta_F A_P}{V_B} I(x_3, x_4)$$

$$L_f h = \nabla h \underline{f} \tag{10}$$
$$= \left(\frac{\beta_F A_P}{V_A} - \frac{\beta_F A_P}{V_B} \right) (-A_P x_2 - C_L(x_3 - x_4))$$

where $L_f h$ is the Lie derivative of the scalar function h with regard to the vector function $\underline{f}$. The Lie derivative and repeated Lie derivatives being defined as:

$$L_f h = \nabla h \underline{f} = \frac{\partial h}{\partial \underline{x}} \underline{f}(\underline{x}) \tag{11}$$

$$L_f^i h = L_f(L_f^{i-1} h) = \nabla(L_f^{i-1} h) \underline{f} \qquad i = 1, 2, ... \tag{12}$$

with $\nabla = \left[\frac{\partial}{\partial x_1} \; \frac{\partial}{\partial x_2} \; ... \; \frac{\partial}{\partial x_n} \right]$ denoting the gradient operator.

Since $L_g h \neq 0$ the control law is selected as:

$$u = \frac{1}{L_g h(\underline{x})} (-L_f h(\underline{x}) + \nu) \tag{13}$$

which yields a force tracking system that converges exponentially to zero, if the desired force trajectory and its first derivative is known. If the control law is substituted into the system it results in the feedback linearised system:

$$\dot{y} = A_P(\dot{x}_3 - \dot{x}_4) = \nu \tag{14}$$

The plant describing the piston position is given by Eq. (5). Hereby the last two equations are feedback linearised by the control law in Eq. (13), and the first two equations, describing the motion of the piston, are linear. Differentiating the second equation of the system yields:

$$\dddot{x}_P = \ddot{x}_2 = \frac{1}{M_{eq}} \left(A_P(\dot{x}_3 - \dot{x}_4) - B_t \dot{x}_2 - \dot{F}_{ext} \right)$$
$$= \frac{1}{M_{eq}} \left(\nu - B_t \dot{x}_2 - \dot{F}_{ext} \right) \tag{15}$$

or:

$$M_{eq} \dddot{x}_P + B_t \ddot{x}_P + K = \nu \tag{16}$$

where K denotes the external forces and disturbances.

Controller Design

The above feedback linearisation yields a system model described by Eq. (16). The control law is selected as:

$$\nu = \hat{M}_{eq}(\dddot{x}_d - k_a \ddot{e} - k_v \dot{e} - k_p e) + \hat{B}_t \ddot{x}_P + \hat{K}$$
$$= \underline{v}^T \underline{\hat{a}} \tag{17}$$

where $\underline{v}^T = [\; \dddot{x}_d - k_a \ddot{e} - k_v \dot{e} - k_p e \quad \ddot{x}_P \quad 1 \;]$ and $\underline{\hat{a}}^T = [\; \hat{M}_{eq} \quad \hat{B}_t \quad \hat{K} \;]$. The constants $\hat{M}_{eq}$, $\hat{B}_t$ and $\hat{K}$ are estimates of the true values. If $\underline{\tilde{a}}$ is defined as $\underline{\hat{a}} - \underline{a}$, Eq. (17) can be rewritten as:

$$M_{eq}(\dddot{e} + k_a \ddot{e} + k_v \dot{e} + k_p e) = \underline{v}^T \underline{\tilde{a}} = \underline{\tilde{a}}^T \underline{v} \tag{18}$$

which, if Laplace transformed yields:

$$e = \frac{1}{s^3 + k_a s^2 + k_v s + k_p} \left[M_{eq}^{-1} \underline{v}^T \underline{\tilde{a}} \right] \tag{19}$$

If $\underline{\hat{a}} = \underline{a} \Rightarrow \underline{\tilde{a}} = 0$ resulting in exponential decay of the error.

The closed-loop error dynamics of Eq. (18) can be described by the following state-space equation:

$$\dot{\underline{x}} = \underline{\underline{A}}\, \underline{x} + \underline{B} \left[\frac{1}{M_{eq}} \underline{\tilde{a}}^T \underline{v} \right]$$
$$e = \underline{C}\, \underline{x} \tag{20}$$

where:

$$\underline{x} = \begin{bmatrix} e \\ \dot{e} \\ \ddot{e} \end{bmatrix}, \underline{\underline{A}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k_a & -k_v & -k_p \end{bmatrix} \tag{21}$$
$$\underline{B}^T = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, \underline{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

When designing the controller the constants k_a, k_v and k_p can be found by pole placement of the system in Eq. (20). Summarised the feedback linearised pressure controller (FLPC) is implemented as:

$$u = \frac{1}{L_g h(\underline{x})} (-L_f h(\underline{x}) + \nu) \tag{22}$$

$$\nu = \hat{M}_{eq}(\dddot{x}_d - k_a \ddot{e} - k_v \dot{e} - k_p e) + \hat{B}_t \ddot{x}_P + \hat{K}$$

where

$$L_g h = \nabla h \underline{g} = \frac{\beta_F A_P}{V_A} H(x_3, x_4) + \frac{\beta_F A_P}{V_B} I(x_3, x_4)$$

$$L_f h = \nabla h \underline{f}$$
$$= -\left(\frac{\beta_F A_P}{V_A} + \frac{\beta_F A_P}{V_B} \right) (A_P x_2 + C_L(x_3 - x_4))$$

Using this controller as a pure state feedback, simulated position tracking errors may be seen in Figure 2. The errors are plotted when applying a sinusiodal reference input for both the two cylinders, making these move from one end position to the other and back in three seconds. To test robustness a mass step of 50 $[kg]$ is applied to the tool center point of the test robot after 6 seconds.

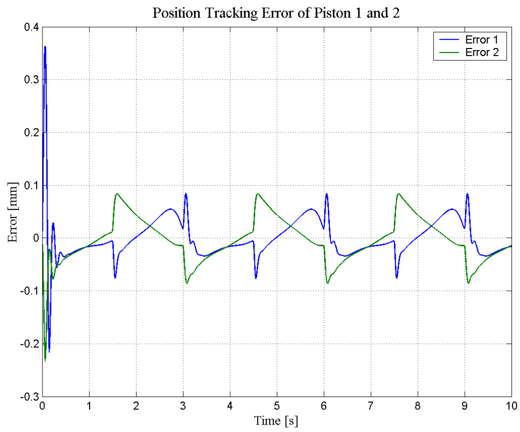

Figure 2 Simulated errors of HSS 1 and HSS 2 using the FLPC. A mass step of 50 $[kg]$ is applied after 6 $[s]$.

A SIMPLIFIED FL CONTROLLER (FLSC)

The strategy derived above requires measurement of the piston acceleration, and in this section a simplified model is derived. This is done by neglecting the pressure dynamics in the cylinder chambers, assuming the pressure build up to be instantaneous, whereby the flow from the servo-valve is set equal to the displacement flow of the cylinder piston. Neglecting the pressure dynamics yields:

$$A_P \dot{x}_P + C_L p_L = \frac{K_v}{\sqrt{2}} u \sqrt{p_S - p_T - \text{sign}(u) p_L} \quad (23)$$

Why the control law u is chosen as:

$$u = \frac{\sqrt{2} A_P (\nu - \lambda p_{Lx})}{K_v \sqrt{p_S - p_T - \text{sign}(u) p_L}} \quad (24)$$

where p_{Lx} is the load pressure (applied by the feedback linearisation) and λ is a scaling factor. Equation (23) can now be written as:

$$A_P \dot{x}_P + C_L p_L = A_P(\nu - \lambda p_{Lx}) \Leftrightarrow \quad (25)$$

$$p_{Lx} = -\frac{A_P \dot{x}_P + C_L p_L - A_P \nu}{A_P \lambda} \quad (26)$$

Substituting the load pressure p_{Lx} into the motion Eq. (1) gives:

$$M \ddot{x}_P = A_P p_{Lx} - B_t \dot{x} - F_L \Rightarrow$$

$$\begin{aligned} \nu &= \frac{M\lambda}{A_P} \ddot{x}_P + \frac{B_t \lambda + A_P}{A_P} \dot{x}_P + \frac{F_L \lambda + C_L p_L}{A_P} \\ &= a_1 \ddot{x}_P + a_2 \dot{x}_P + a_3 \end{aligned} \quad (27)$$

If the theory, used in the previous section, is applied to the this second-order system, the control law becomes:

$$\nu = \hat{a}_1(\ddot{x}_d - k_v \dot{e} - k_p e) + \hat{a}_2 \dot{x}_P + \hat{a}_3 = \underline{v}^T \underline{\hat{a}} \quad (28)$$

where $\underline{v}^T = [\ddot{x}_d - k_v \dot{e} - k_p e \quad \dot{x}_P \quad 1]$, $\underline{\hat{a}}^T = [\hat{a}_1 \quad \hat{a}_2 \quad \hat{a}_3]$ and the constants $\hat{a}_1$, $\hat{a}_2$ and $\hat{a}_3$ are estimates of the true values.

This results in a system, where the closed-loop error dynamics can be described by the following state-space equation:

$$\begin{aligned} \dot{\underline{x}} &= \underline{\underline{A}}\, \underline{x} + \underline{B} \left[\frac{1}{a_1} \underline{\tilde{a}}^T \underline{v} \right] \\ e &= \underline{C}\, \underline{x} \end{aligned} \quad (29)$$

with:

$$\underline{x} = \begin{bmatrix} e \\ \dot{e} \end{bmatrix}, \underline{\underline{A}} = \begin{bmatrix} 0 & 1 \\ -k_v & -k_p \end{bmatrix} \quad (30)$$
$$\underline{B}^T = \begin{bmatrix} 0 & 1 \end{bmatrix}, \underline{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Summarised the controller is implemented as:

$$u = \frac{\sqrt{2} A_P (\nu - \lambda p_{Lx})}{K_v \sqrt{p_S - p_T - \text{sign}(u) p_L}} \quad (31)$$

where

$$\nu = \hat{a}_1(\ddot{x}_d - k_v \dot{e} - k_p e) + \hat{a}_2 \dot{x}_P + \hat{a}_3$$

Adjusting the controller parameters based on pole placement, with the desired pole locations being $\sigma = -100 \pm 40i$, simulated position tracking errors are shown in Figure 3.

Experimental Results of the FLSC

To verify the performance of the controller, this is implemented and tested on the laboratory setup. Implementation of the algorithm showed sensitivity to noise in both the position and pressure signals, why both of these had to be filtered using second-order digitally implemented filters. As a consequnce hereof the controller parameters needed to be changed, so the poles were placed as $\sigma = -20 \pm 20i$, which degraded the performance of the controller. Experimental measurements using this controller are shown in Figure 4.
Comparing the simulated and the experimantal results it is seen that the measured tracking errors are approximately an order of magnitude larger than the simulated due to the readjusted controller parameters.

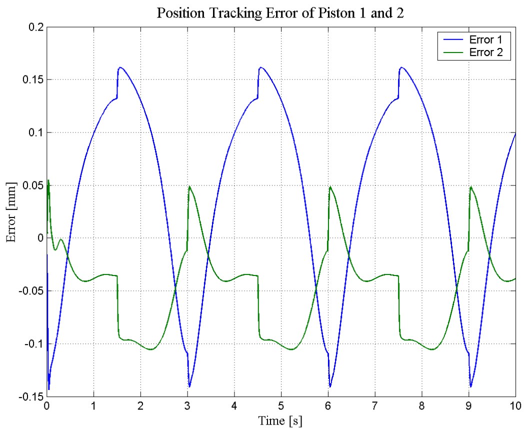

Figure 3 Simulated errors of HSS 1 and HSS 2 using the FLSC.

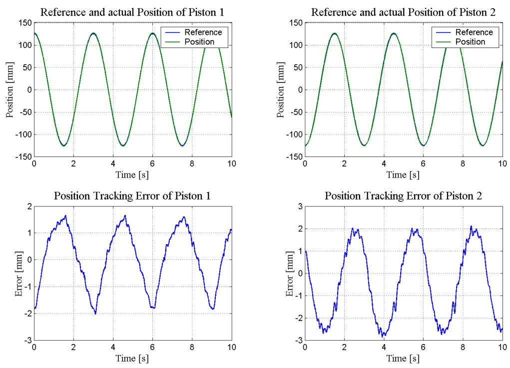

Figure 4 Errors of HSS 1 and HSS 2 using the FLSC on the laboratory setup.

ADAPTIVE FEEDBACK LINEARISED SECOND-ORDER CONTROLLER (AFLSC)

In the previous section the system parameters are estimated and consequently the controller the parameters are adjusted conservatively. In order to compensate for these parameter variations an adaptive controller has been implemented. For the error equation (29), a standard adaptive control law including both an "integral" and a "proportional" part is chosen to:

$$\dot{\underline{\hat{a}}} = -\mathrm{sign}(a_1^{-1})\gamma e\underline{v} - \mathrm{sign}(a_1^{-1})\alpha\frac{d}{dt}(e\underline{v}) \quad , \quad \gamma, \alpha > 0 \tag{32}$$

The controller with adaptation capabilities to account for unmodelled dynamics is thus implemented as:

Control law (AFLSC):

$$u = \frac{\sqrt{2}A_P(\nu - \lambda p_{Lx})}{K_v\sqrt{p_S - p_T - \mathrm{sign}(u)p_L}} \tag{33}$$

where

$$\nu = \hat{a}_1(\ddot{x}_d - k_v\dot{e} - k_p e) + \hat{a}_2\dot{x}_P + \hat{a}_3$$

Adaptation law (AFLSC, expanded):

$$\begin{bmatrix} \dot{\hat{a}}_{1q} \\ \dot{\hat{a}}_2 \\ \dot{\hat{a}}_3 \end{bmatrix} = \begin{bmatrix} \dot{\hat{a}}_1 \\ \dot{\hat{a}}_2 \\ \dot{\hat{a}}_3 \end{bmatrix} = \begin{bmatrix} c_1 \\ -\gamma_2 e\dot{x}_P - \alpha_2\frac{d}{dt}(e\dot{x}_P) \\ -\gamma_3 e - \alpha\frac{d}{dt}(e) \end{bmatrix}$$

with $c_1 = -\gamma_1 e(\ddot{x}_d - k_v\dot{e} - k_p e) - \alpha_1\frac{d}{dt}(e(\ddot{x}_d - k_v\dot{e} - k_p e))$.

A schematic illustration of the implementation is shown in fig. 5. Simulation results for this controller are shown in fig. 6 and experimental results are given in fig. 7.

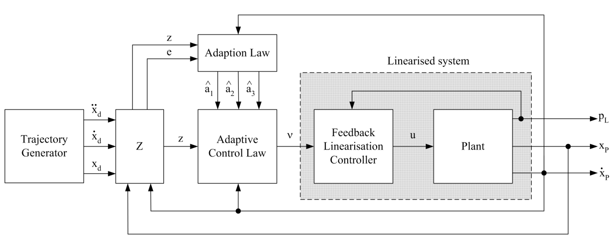

Figure 5 Block diagram of the system using the AFLSC.

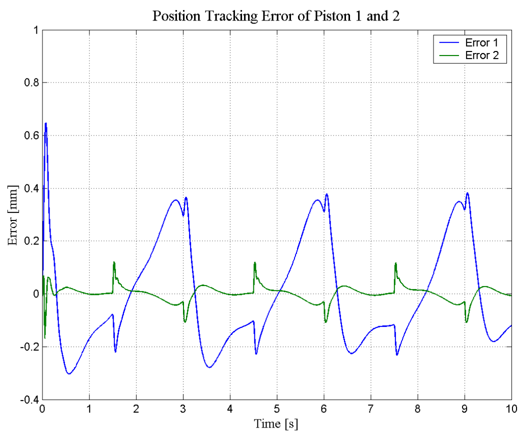

Figure 6 Simulated errors of HSS 1 and HSS 2 using the AFLSC.

As seen from the results the performance of the AFLSC is very good and the steady-state errors on both cylinder

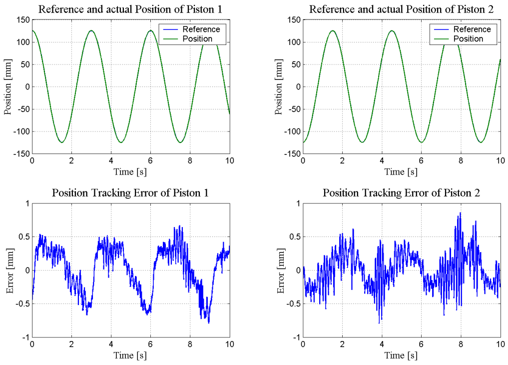

Figure 7 Errors of HSS 1 and HSS 2 using the AFLSC on the laboratory setup.

pistons are limited to ± 0.5mm. From the position tracking errors, shown in fig. 7, it may be seen that it is very chattered, due to the velocity signal being obtained by differentiating the position signal and filtering.

CONCLUSIONS

The focus of this paper has been the development and comparison of several feedback linearisation controllers applied on a hydraulic servo robot. In the first controller (FLPC) a part of the model was linearised, namely the pressure dynamics. This linearisation results in a linear control system, but with varying inertia, which showed good performance both in the case of varying parameters and robustness to load changes. The disadvantages of the controller is the number of needed state measurements. The second controller (FLSC) was based on a model reduced to a second-order system, by neglecting the pressure dynamics. The controller showed very good performance in the simulation, but was sensitive to noise in the laboratory, why the controller parameters had to be retuned, decreasing the performance considerably.

To improve the performance of the FLSC, the AFLSC was developed by adding an adaptive control scheme, whereby the performance was improved significantly. Though the AFLSC is fairly complex, the controller parameters are easily adjusted. Common for the controllers considered are that they are very robust to mass variations, as a stepwise increase in the tool centre point mass by 50 $[kg]$ has no significant influence on the resulting position tracking error response.

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