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A Comparative Study of Modelling Techniques for Laminar Flow Transients in Hydraulic Pipelines

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ABSTRACT

Accurate predictions of pressure surges are crucial in many fluid-line systems. There are several techniques available for modelling flow transients in hydraulic pipelines, offering different advantages and disadvantages and therefore suiting different applications. In this paper four modelling methods are evaluated in terms of accuracy, computational efficiency and flexibility for laminar, single-phase flow in circular, rigid pipes. These are the method of characteristics (MOC), the finite element method (FEM), the transmission line method (TLM), and the rational polynomial transfer function approximation (RPTFA) method, which is a form of modal approximation method. All four methods were implemented in MATLAB/Simulink. It was found that the RPTFA model potentially gives the most accurate solution, while the MOC and TLM models are the most computationally efficient. The FEM is the least accurate out of the four methods, but it can be used with varying parameters and time steps thus providing the most modelling flexibility.

KEYWORDS

Fluid transients, Fluid transmission lines, Laminar flow

NOMENCLATURE

A	Cross-sectional area
$\mathbf{B}$	Coefficient matrix
c	Speed of sound
$\mathbf{C}$	Coefficient matrix
C	Laplace transformed characteristic
$\mathbf{E}$	Coefficient matrix
f	Friction term
$\mathbf{F}$	Coefficient matrix
p	Pressure
$\mathbf{p}$	Pressure vector
P_1	Pressure at upstream end
P_2	Pressure at downstream end
q	Flow
$\mathbf{q}$	Flow vector
Q_1	Flow at upstream end
Q_2	Flow at downstream end
Z	Characteristic impedance
Z_0	Characteristic impedance constant
Γ	Propagation operator
ρ	Fluid density

INTRODUCTION

Hydraulic systems are widely used in the engineering world. They often form an essential part of a total dynamic system in industries such as automotive,

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aerospace, petroleum, etc. As these systems become more sophisticated, accurate analysis and close control of their performance becomes ever more crucial. This in turn may require consideration of the dynamics of the hydraulic fluid transmission lines which can have a significant influence on system behaviour.

Transient changes in flow can generate pressure waves that travel through the body of the fluid in the transmission lines. This dynamic behaviour can be represented by the equations of motion and continuity, as follows (assuming $q / A << c$):

$$\frac{\partial q}{\partial t} + \frac{A \partial p}{\rho \partial x} + f(q) = 0 \quad (1)$$

$$\frac{\partial p}{\partial t} + \frac{\rho c^2}{A} \frac{\partial q}{\partial x} = 0 \quad (2)$$

Hydraulic pipeline dynamics are well understood in the frequency domain. However, obtaining the transient response of the system in the time domain requires approximating the hyperbolic functions which represent the frequency-dependent component of friction, $f(q)$.

There are several techniques available for solving equations (1) and (2) and approximating friction in the time domain. The four most established modelling methods are the method of characteristics [1], the transmission line method [2], the modal approximation method [3], and the finite element method [4]. It is beyond the scope of this paper to present and discuss these in detail. Here, these four methods are evaluated in terms of accuracy, computational efficiency and flexibility. To this end, laminar flow transients in a rigid, circular pipeline were simulated in MATLAB. The system under investigation comprised a constant pressure source upstream and a valve downstream of the pipe. Flow transients were generated by instantaneous valve closure.

METHOD OF CHARACTERISTICS

In the method of characteristics (MOC) the equations of motion and continuity are transformed into ordinary differential equations which can then be integrated along the 'characteristic lines'. The characteristic lines correspond to the motion of waves travelling at the speed of sound in both directions along a pipe. Friction in pipelines is frequency-dependent and its accurate and correct representation poses one of the biggest challenges in transient flow modelling. Although the steady-state friction component is well defined, the unsteady friction component can only be approximated for time-domain simulation. Initial work by Zielke [5] provided an accurate solution, but at the expense of a very high computational load. Trikha [6] and Kagawa et al. [7] later evaluated weighting factors for a series of unsteady friction terms, in an attempt to simplify computations. A set of optimised coefficients was obtained by Taylor et al. [8] reducing at the same time the number of unsteady friction terms from five in Trikha's model to four.

All three unsteady friction approximations mentioned above were used to simulate the transient response of the system to an instantaneous valve closure. The exact solution that was obtained from an inverse Fourier transform of the analytical response is plotted in figure 1, along with the steady-state friction. The three error traces are also plotted for comparison. Johnston [9] recently proposed an alternative method of selecting the weighting factors using a geometric series. The number of terms needed depends on the time step, but also on the value of the geometric series ratio. It was found that a model with a ratio of 3 and three terms gives results almost identical to those obtained using the Taylor et al. and Kagawa et al. approximations, see figure 1.

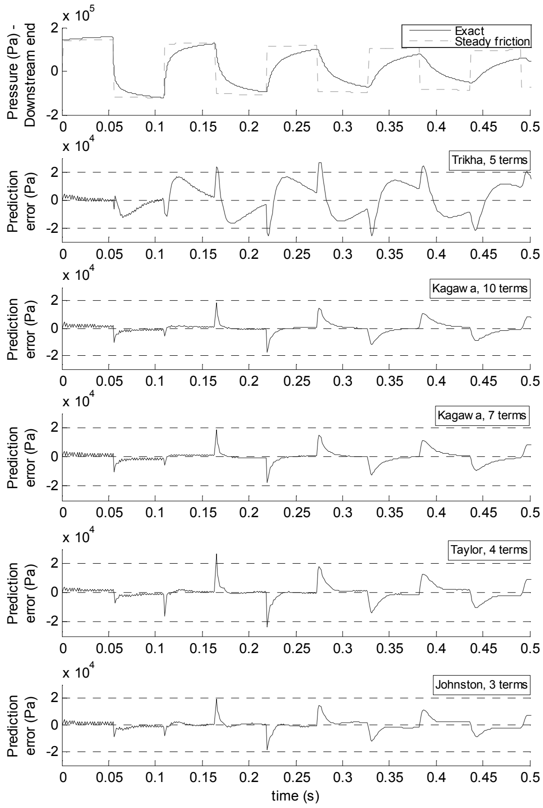

Figure 1: Predicted transient response, MOC, different friction models

In all the above simulations the friction terms are calculated at every node along the pipe and at every timestep. In all simulations the pipeline was divided into 20 elements. To reduce computation load, the

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friction terms can be lumped together at the end of the pipe. Two cases were considered. First, the frequency-dependent friction was lumped at the pipeline ends, while the steady-state friction was evenly distributed along the pipe. In the second case, both friction components were lumped at the ends of the pipe during simulation. The prediction errors, compared to the exact solution shown in figure 1, are plotted in figure 2. A very similar response was obtained in both cases. A 77.4% reduction in computational time was achieved in the first case, and an 87.2% in the second, at the expense of a slight loss in prediction accuracy.

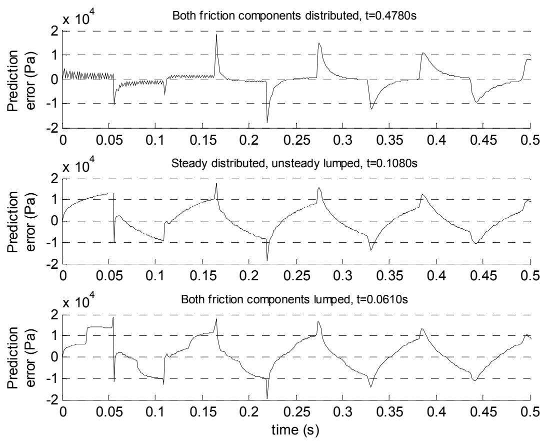

Figure 2: Predicted transient response, MOC, lumped friction

TRANSMISSION LINE METHOD

The transmission line method (TLM) is similar to the MOC. In the MOC a pipeline is divided into a number of elements and pressure and flow propagates between adjacent nodes over one timestep. In TLM, the model only calculates these variables at the ends of a line and not at any internal nodes. Obtaining the correct steady state pressure drop and rate of oscillation decay, however, requires careful modelling. Krus et al. [2] proposed a TLM model where filters are used to approximate the frequency-dependent friction. The system response can then be obtained by introducing characteristics C_1 and C_2 such that

$$P_1 = C_1 + \frac{\rho c}{A} Q_1 \quad (3)$$

$$P_2 = C_2 + \frac{\rho c}{A} Q_2 \quad (4)$$

The Simulink block diagram of the model is shown in figure 3. The frequency and shape of the predicted pressure waves were found to be very inaccurate. This is shown in figure 4, where the TLM prediction is compared with that obtained using the MOC. Johnston [9] obtained much improved results by incorporating his friction model into Krus et al.'s model. The predictions obtained using both a three- and a four-term Johnston friction model are also plotted in figure 4 for comparison.

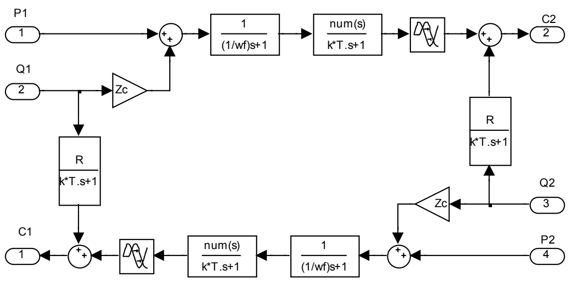

Figure 3: Transmission line model

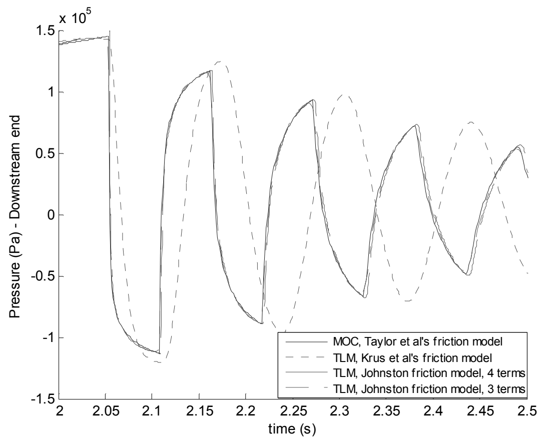

Figure 4: Predicted transient response, TLM and MOC

MODAL APPROXIMATIONS

The dynamics of a fluid transmission line can be expressed in terms of a so-called 'dissipative' distributed-parameter model using the following input-output relationship:

$$\begin{bmatrix} P_2 \\ Z_0 Q_1 \end{bmatrix} = \begin{bmatrix} \dfrac{1}{\cosh\Gamma} & -\dfrac{Z\sinh\Gamma}{Z_0\cosh\Gamma} \\ \dfrac{Z_0\sinh\Gamma}{Z\cosh\Gamma} & \dfrac{1}{\cosh\Gamma} \end{bmatrix} \begin{bmatrix} P_1 \\ Z_0 Q_2 \end{bmatrix} \quad (5)$$

The line characteristic impedance Z and the propagation operator Γ are functions of the fluid properties and the radius and length of the line. Obtaining an exact solution involves zero- and first-order Bessel functions. Several modal approximation techniques have been proposed over the years for the representation of the

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three transfer functions (TF) of equation (5). Here, a recently proposed method by Wongputorn et al. [3] was used which utilises least-squares curve fitting in the frequency domain to match the frequency response of the original TFs to that of rational polynomial TFs over the frequency range of interest.

When the rational polynomial transfer function approximation (RPTFA) method is used, it is very important to ensure that the transmission line model is very accurate throughout the modal frequencies of the other components of the system in which the line is an internal part and beyond any input disturbance frequencies, [3]. Different RPTFAs were obtained over different frequency ranges by defining the maximum desired frequency. The latter was varied between 100 rad/s and 2000 rad/s. At 1500 rad/s the TF approximations become inaccurate, and at 2000 rad/s the solution becomes unstable. The frequency response of one of the three TFs needed to model the system and its approximation, as obtained from Wongputorn's MATLAB routine [10], is shown in figure 5. The plotted approximation was obtained for a maximum frequency of 1000 rad/s.

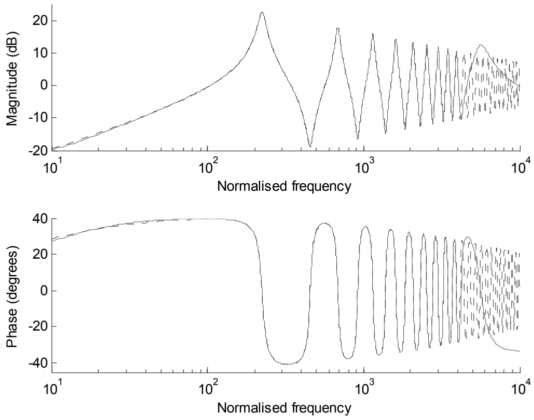

Figure 5: Frequency response of transfer function ' $Z \sinh\Gamma / Z_0 \cosh\Gamma$ ' and its approximation

The RPTFA model, incorporating transfer function approximations for frequencies up to 1000 rad/s, was implemented in Simulink. Because the system is mathematically stiff, the choice of ordinary differential equation (ODE) solver in the Simulink model is critical and an accurate solution cannot always be obtained. The solution often becomes unstable or very inaccurate, depending both on the approximation functions and the solver used.

However, when good results were obtained using RPTFA they were found to be more accurate than the solution obtained with the MOC, as illustrated in figure 6. The phase shift between the exact and the MOC solution, which develops as the wave attenuates, is no longer present in the RPTFA solution. Inaccuracies in the solver, however, make the RPTFA solution oscillate around the exact signal throughout the simulation. This behaviour is more evident in the early part of the solution. The response of the system obtained using the RPTFA method is compared with that obtained using the MOC and the TLM techniques in figure 7.

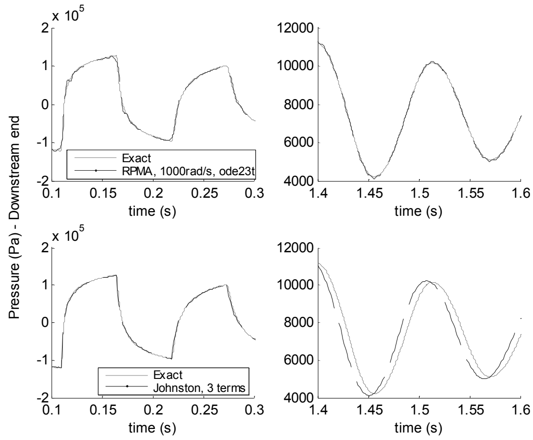

Figure 6: Comparison between MOC and RPTFA methods

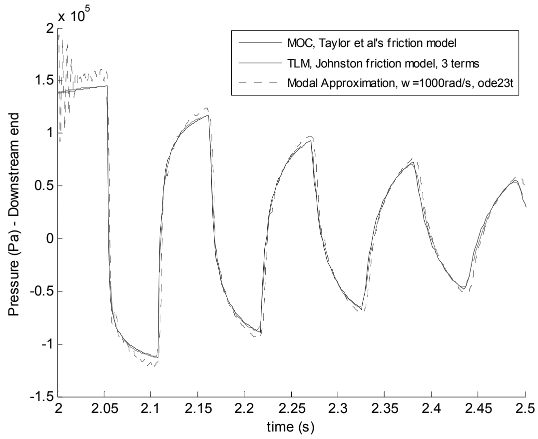

Figure 7: Comparison between MOC, TLM and RPTFA methods

Provided that a stable numerical solution can be obtained using the available MATLAB solvers, the RPTFA model is the most accurate of all the models considered so far. One of its main drawbacks, however, is that numerical solutions are often unstable or oscillatory. To overcome this problem, more appropriate solvers for this application could be implemented in MATLAB. Alternatively, different modal approximation methods that are available in the literature could be considered [11-13].

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FINITE ELEMENT METHOD

Finite element approximations to the equations of motion and continuity may be derived according to:

$$\frac{d\boldsymbol{q}}{dt} = \boldsymbol{Bp} + \boldsymbol{Cf}(\boldsymbol{q}) \quad (6)$$

$$\frac{d\boldsymbol{p}}{dt} = \boldsymbol{Eq} + \boldsymbol{F\overline{q}} \quad (7)$$

Here, the finite element method (FEM) proposed by Sanada et al. [4] and later extended by Taylor et al. [8] was chosen to simulate the transient response of the transmission line. The FEM model uses an interlaced and unequally spaced grid of alternate pressure and flow nodes. A genetic algorithm was applied by the authors to optimise the node spacing. Grid points are spaced symmetrically about the mid point of the pipeline. The optimisation algorithm minimises the error between the natural frequencies of the model and those of a specific pipeline for the extreme cases of closed and open end conditions. The model incorporates the four-term laminar friction model proposed by Johnston [9].
When the optimised grid is used, the model exhibits a highly oscillatory behaviour during the first moments following the transient. Less severe oscillations are present when the equally spaced grid is used. These die out as the pressure wave attenuates and the shape of the pulsations for the optimised grid is better after the first two cycles. As can be seen from figure 8, the period of the pressure wave is very slightly different for the two cases.

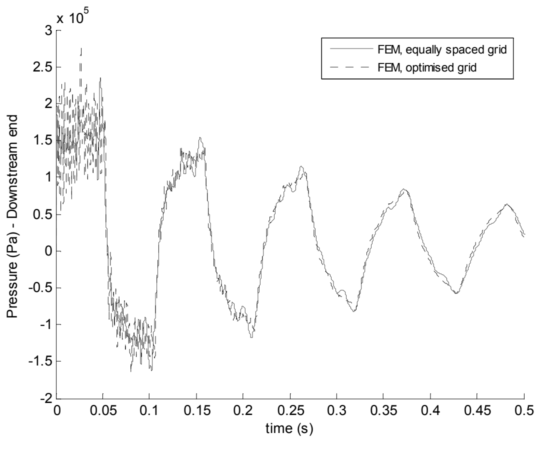

Figure 8: Comparison between optimised and equally spaced grid, FEM

The transient response of the FEM model (40 elements) is compared with those obtained using the three modelling techniques presented above, namely MOC, TLM, and RPTFA, in figure 9. While the prediction of the RPTFA model is the one closest to the exact solution, the prediction obtained using the FEM is the least accurate one. However, the FEM was found to be reliable and not prone to instability.

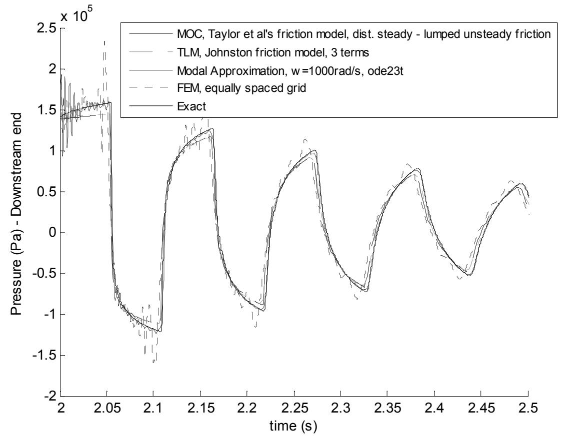

Figure 9: Comparison between MOC, TLM, RPTFA, and FEM methods

COMPUTATIONAL EFFICIENCY

The performance of the different modelling techniques was compared in terms of computational efficiency. The most accurate models developed using the four different methods were chosen for the comparison. The transient response of the system was simulated on a 2.4 GHz Pentium 4 PC with 1 Gb of RAM, for 2 seconds following a sudden valve closure. The results are summarised in the table below.

Table 1 Computational times

MOC

No. of elements	Friction model	No. of terms	Steady state friction	Unsteady friction	Computational time (s)
20	Taylor et al.	4	distributed	distributed	0.4680
20	Kagawa et al.	7	distributed	distributed	0.4850
20	Johnston	3	distributed	distributed	0.4530
20	Johnston	3	distributed	lumped	0.0790
20	Johnston	3	lumped	lumped	0.0630

TLM

Friction model	No. of terms	MATLAB Solver	Time step	Step size	Computational time (s)
Taylor et al.	4	ode4	fixed	10^{-4}	1.2240
Johnston	4	ode4	fixed	10^{-4}	1.2645
Johnston	3	ode4	fixed	10^{-4}	1.0850
Johnston	3	ode45	variable	10^{-3} (max)	0.1845
Johnston	3	ode45	variable	-	0.0700
Johnston	4	ode45	variable	-	0.4385

RPTFA

Frequency - max, (rad/s)	MATLAB Solver	Time step	Computational time (s)	Notes
1000	ode23t	variable	0.6955	Solution oscillates around exact response
800	ode23tb	variable	0.8545	
500	ode23t	variable	0.9475	

FEM

No. of elements	Friction model	No. of terms	Grid spacing	Computational time (s)
40	Johnston	4	uniform	2.521
80	Johnston	4	uniform	13.148
160	Johnston	4	uniform	180.866

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Obtaining the response of the RPTFA model using the MATLAB solvers often leads to unstable or oscillatory solutions. Oscillatory solutions can either diverge from or converge to the exact system response. The increased computational times shown in the last two rows of the RPTFA table can be mainly attributed to the high frequency oscillation of the model output around the exact system response. Use of a more accurate and robust integrator may reduce computational times when the RPTFA model is used and help obtain more stable solutions.

CONCLUSIONS

A preliminary investigation of the performance of four different transient flow modelling techniques has been completed. These techniques are the method of characteristics, the transmission line method, the finite element method, and the modal approximation method. The RPTFA model was found to give the most accurate solutions. Computational times, however, are 10 times longer than those when the MOC model is used and an accurate solution cannot always be obtained with the existing MATLAB solvers. The MOC model is best suited to a fixed time step, but accurate results in short computational times can be obtained when the friction components are lumped at the pipe ends. Applying the TLM model provides both an accurate and computationally efficient method of simulating the response of the system, but integration problems can occur. The FEM method is the least accurate and efficient in terms of computational time, but can handle non-linearities and varying parameters and time steps enabling thus the modelling of cavitation and air-release.

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REFERENCES

1. Fox J.A., Hydraulic Analysis of Unsteady Flow in Pipe Networks, The Macmillan Press Ltd., 1977.
2. Krus P., Weddfelt K., Palmberg J.O., Fast Pipeline Models for Simulation of Hydraulic Systems, Transactions of the ASME, Journal of Dynamic Systems, Measurements, and Control, March 1994, Vol. 116, No. 1, pp. 132-136.
3. Wongputorn P., Hullender D.A., Woods R.L., Rational Polynomial Transfer Function Approximations for Fluid Transients in Lines, Proceedings of ASME FEDSM 2003, 4th ASME_JSME Joint Fluids Engineering Conference, Honolulu, Hawaii, July 2004, pp. 2797-2804.
4. Sanada K., Richards C.W., Longmore D.K., Johnston D.N., A Finite Element Model of Hydraulic Pipelines Using an Optimized Interlacing Grid System, Proceedings of Institution of Mechanical Engineers, Journal of Systems and Control Engineering, Vol. 207, pp. 213-220, 1993.
5. Zielke W., Frequency-Dependent Friction in Transient Pipe Flow, Transactions of the ASME, Journal of Basic Engineering, March 1968, pp. 109-115.
6. Trikha A.K., An Efficient Method for Simulating Frequency-Dependent Friction in Transient Liquid Flow, Transactions of the ASME, Journal of Fluids Engineering, March 1975, pp. 97-105.
7. Kagawa T., Lee I., Kitagawa A., Takenaka T., High Speed and Accurate Computing of Frequency-Dependent Friction in Laminar Pipe Flow for Characteristics Method, JSME, Vol. 49, No. 447, pp. 2638-2644, 1993.
8. Taylor S.E.M., Johnston D.N., Longmore D.K., Modelling of Transient Flow in Hydraulic Pipelines, Proceedings of Institution of Mechanical Engineers, Vol. 211, pp. 447-456, 1997.
9. Johnston D.N., Efficient Methods for Numerical Modelling of Laminar friction in Fluid Lines, Proceedings of IMECE 2004, 2004 ASME International Congress and R&D Expo, November 2004, Anaheim, CA, USA.
10. Internet website for downloading fluid transmission line MATLAB m-files written by D.A. Hullender: maepro.uta.edu/Hullender
11. Kojima E., Shinada M., Yu J., Development of Accurate and Practical Simulation Technique Based on the Modal Approximations for Fluid Transients in Compound Fluid-Line Systems, International Journal of Fluid Power 3, No.2, pp. 5-15, 2002
12. Yang W.C., Tobler W.E., Dissipative Modal Approximation of Fluid Transmission Lines Using Linear Friction Model, Transactions of the ASME, Journal of Dynamic Systems, Measurements, and Control, March 1991, Vol. 113, pp. 152-162
13. Hsue C. Y-Y., Hullender D.A., Modal Approximations for the Fluid Dynamics of Hydraulic and Pneumatic Transmission Lines, Proceedings of ASME Winter Annual Meeting on Fluid Transmission Line Dynamics, Boston, 1983, pp. 51-77

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